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THE TRANSITION CURVE.

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THE TRANSITION CURVE

BY OFFSETS AND BY DEFLECTION ANGLES.

BY

C. L. CRANDALL, C.E.,

Professor of Railway Engineering, Cornell University;
Member American Society of Civil Engineers.

SECOND EDITION, REVISED AND ENLARGED.

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PREFACE.

MUCH has been written upon the subject of transition curves within the past few years, and the theory and practice have been accurately worked up for small central angles, notably in the "Holbrook Spirals." Accurate methods are here, it is believed for the first time, developed for the true transition curve, curvature increasing directly with the distance, which will hold for large central angles for both the offset and the deflection methods. The extension of the limits to include long transition curves with large central angles increases the flexibility of the alignment of a railroad, which will tend to economy of construction in all but the most easy country.

The offset method is worked up for use during location and the greater part of construction on account of its extreme simplicity; the increased labor over that required for simple curves being almost insignificant.

The method by deflections with transit and chain will usually be preferred for curves over 200 to 300 feet in length, in running centers on the finished roadbed, for laying track, or other accurate work. Table III will give the deflections for any true transition curve connecting a circular curve with a tangent, or connecting the two branches of a compound curve, with accuracy for all central angles of transition curve up to 60°. The deflections taken out for the most common curves can be conveniently tabulated on the blank pages inserted for the purpose.

While intended primarily for the use of civil engineering students, it is believed that the complete set of tables given will render the methods at least as rapid and convenient in actual use in the field as the more restricted or approximate ones now in use.

Special acknowledgment is due to H. N. Ogden, C.E., formerly an instructor here, for carefully revising the text.

CORNELL UNIVERSITY,
ITHACA, N. Y., Jan., 1893.

PREFACE TO THE SECOND EDITION.

On account of the increased use of transition curves for electric railroads, Table I, containing offsets and other transition curve data, has been extended to circular curves of short radii.

Table IV, supplementing Table III with actual deflections in degrees and minutes for a large range of transition curves, has been added. This table in its general form and in the range of curves given was suggested by the blue-print tables in use on the Burlington and Missouri River Railroad, and kindly loaned by E. E. Hart, C.E., a former resident engineer. When the degree of the circular curve and the length of the transition can be found in the table without interpolation, the table is more convenient than Table III.

A five-place table of sines, cosines, tangents, and cotangents has been added, so that the simple curve problems involving transition curves can be worked in the field without the use of the regular field-book.

Cornell University,
Ithaca, N. Y., Dec., 1898.

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TRANSITION CURVES.

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1. Necessity for Transition Curves.—The road-bed upon a tangent or straight track is made level transversely; upon a circular curve it is inclined toward the center to counteract centrifugal force. The change from the one to the other must be gradual for easy riding, and to prevent twisting the trucks. Hence if the inclination of the road-bed is to be proportional to the centrifugal force at every point, a transition curve must be used, in which the curvature increases directly with the distance, in passing from the tangent to the circular curve.

2. Different Kinds in Use.—Rankine (*Civil Engineering*) gives the credit of first laying out railroad curves with a gradual increase of curvature, starting from the tangent, to Mr. Gravatt about 1828; of using such a curve to connect the tangent and main circular curve, to Mr. Wm. Froude about 1842,—their methods being first published in 1860-1.

In this country transition curves have come into use within the last ten to twelve years. The true transition curve, curvature increasing directly with the distance, was described in *The Railroad Gazette*, Dec. 3, 1880, by Ellis Holbrook. *The Railroad Spiral*, a curve made up of circular arcs compounded, by W. H. Searles, was published in 1882. The cubic parabola, in which the curvature increases with the distance along the tangent, was described by C. D. Jameson and E. W. Crellin in the *Railroad and Engineering Jour.* in 1889. The Froude curve, as worked up by A. M. Wellington, was described in the *Engineering News* in 1890. These bring out the more important classes, although the list is far from complete.

All but the last were worked out to be run with transit and chain, the deflections being given for several curves with

different rates of increase of curvature for flexibility; the last was worked up to be run with transit, or by ordinates from the circular curve and from the tangent, as preferred. It is the only one of the list which is worked up for a curve of any length or of any offset from the tangent to the main circular curve; the field-work during location is reduced to a minimum by offsetting from the tangent directly to the circular curve, but the fundamental assumptions become inconsistent when the curve includes a large central angle.

THEORY.

3. Equation of Curve.—In Fig. 1, let EGE' be a circular curve with centre C and tangents EK and $E'K$. It is

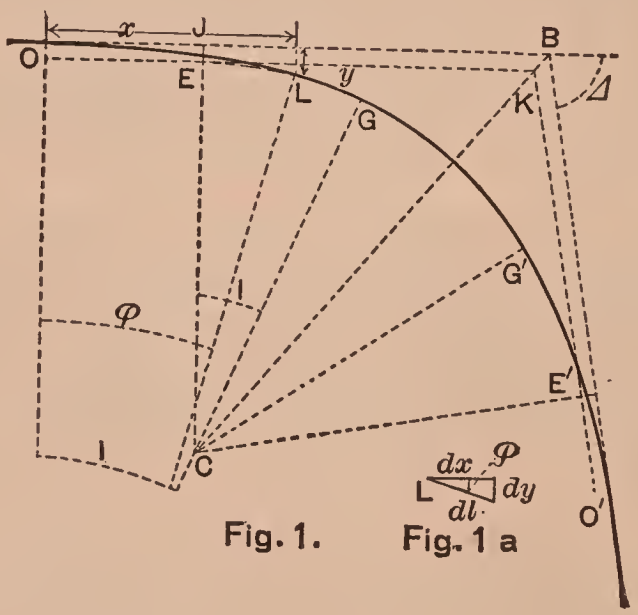

Fig. 1. Fig. 1 a

evident that if we begin at G to reduce the curvature directly with the distance, and continue the reduction until the curvature is zero, using the same central angle I as for the corresponding portion of the circular curve, the new curve will pass outside the old, requiring a tangent OB parallel with KE, and at some distance as JE from it.

E is called the $P.C.$, as usual; O, the $P.T.C.$; G, the $P.C_1.$; G', the $P.T.C_1.$; E', the $P.T.$; O', the $P.T_1.$

Since the curvature is zero at O and increases directly with the distance, the ratio of the curvature and distance increments, $d\phi$ and dl, must increase with l, i.e.,

$$\frac{d\phi}{dl} = 2kl, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $2k$ is a constant for any one curve, but varies for different curves.

Integrating,

$$\phi = kl^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ϕ is the central angle up to the point L, l the length, and k a constant.

In the right differential triangle, Fig. 1*a*,

$$dy = dl \sin \phi$$
$$= dl \sin kl^2, \quad \text{since} \quad \phi = kl^2 \text{ by (2).}$$

The expansion of the sine in terms of the angle, by Trigonometry, is

$$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} +,$$

where ! denotes a factorial number, i.e., $3! = 1 \times 2 \times 3 = 6$, etc.

Substituting,

$$dy = dl\left(kl^2 - \frac{k^3l^6}{3!} + \frac{k^5l^{10}}{5!} - \frac{k^7l^{14}}{7!} +\right).$$

Integrating,

$$y = \frac{kl^3}{3} - \frac{k^3l^7}{42} + \frac{k^5l^{11}}{1320} - \frac{k^7l^{15}}{75600} + . \quad . \quad . \quad . \quad . \quad (3)$$

Restoring ϕ for kl^2,

$$y = \frac{l\phi}{3} - \frac{l\phi^3}{42} + \frac{l\phi^5}{1320} - \frac{l\phi^7}{75600} +,$$

where ϕ is in π-measure, coming from the development of the sine in series.

For ϕ in degrees $\left(\phi = \phi^\circ \frac{\pi}{180} = .017453\phi^\circ\right)$,

$$y = l\left[\tfrac{1}{3}(.017453)\phi^\circ - \tfrac{1}{42}(.017453)^3\phi^{\circ 3} + \tfrac{1}{1320}(.017453)^5\phi^{\circ 5} - \tfrac{1}{75600}(.017453)^7\phi^{\circ 7} + \right],$$

or
$$y = lC, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where C varies with ϕ°; it can be taken from Table II, with ϕ° as argument.

From Fig. 1*a*,

$$dx = dl \cos \phi$$
$$= dl \cos kl^2, \quad \text{since} \quad \phi = kl^2 \text{ by (2).}$$

The expansion of the cosine in terms of the angle, by Trigonometry, is

$$\cos t = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + .$$

Substituting,

$$dx = dl\left(1 - \frac{k^2l^4}{2!} + \frac{k^4l^8}{4!} - \frac{k^6l^{12}}{6!} + \right).$$

Integrating,

$$x = l - \frac{k^2l^5}{10} + \frac{k^4l^9}{216} - \frac{k^6l^{13}}{9360} + . \quad . \quad . \quad . \quad . \quad (5)$$

Restoring ϕ for kl^2,

$$x = l - \frac{l\phi^2}{10} + \frac{l\phi^4}{216} - \frac{l\phi^6}{9360} + .$$

For ϕ in degrees,

$$x = l - l\left[\tfrac{1}{10}(.017453)^2\phi^2 - \tfrac{1}{216}(.017453)^4\phi^4 + \tfrac{1}{93600}(.017453)^6\phi^6 - \right],$$

or
$$x = l - lE, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where E varies with ϕ°; it can be taken from Table II with ϕ° as argument.

4. Coördinates of Center.—Since the increment to the central angle, $d\phi$, is in π-measure, = arc dl divided by radius of curvature ρ, we have from (1),

$$\frac{dl}{d\phi} = \rho, \quad = \frac{1}{2kl}. \quad \ldots\ldots \quad (7)$$

At the $P.C_1.$, $\rho = R$, the radius of the circular curve; $\phi = I$, the central angle of the circular curve from $P.C.$ to $P.C_1$.

Therefore, from (2) and (7),

$$\left.\begin{aligned} I &= kl_1^2, \quad = \frac{l_1}{2R} \\ \phi &= I\frac{l^2}{l_1^2} \end{aligned}\right\}, \quad \ldots\ldots \quad (8)$$

where l_1 = the length of the transition curve to $P.C_1$.

But $R = \frac{5730^*}{D}$; $I^\circ = I\frac{180}{\pi} = 57.30I$.

$$\therefore I^\circ = \frac{l_1 D}{200}, \quad = 28.65\frac{l_1}{R}, \quad \ldots\ldots \quad (9)$$

$$k = \frac{1}{2Rl_1} = \frac{D}{11460l_1}. \quad \ldots\ldots \quad (10)$$

If x_1 and y_1 are the coördinates of $P.C_1.$, and x', F, those of the $P.C.$ (see Fig. 2),

$$\left.\begin{aligned} F &= y_1 - R(1 - \cos I) \\ x' &= x_1 - R\sin I \end{aligned}\right\}, \quad \ldots\ldots \quad (11)$$

while the coördinates of the center will be x' and $F + R$.

5. Deflection Angles.—With the instrument at station O, Fig. 2 ($P.T.C.$, or $P.T_1.$), the tangent of the deflection angle d_0 can be found by dividing y by x, using the values from equations (3) and (5), thus:

$$\tan d_0 = \frac{y}{x} = \frac{kl^2}{3} + .009\,523k^3l^6 + .000\,167k^5l^{10} + .$$

* If curves from 8° to 16° are run with 50-foot chords, from 16° to 32° with 25-foot chords, from 32° to 80° with 10-foot chords, and all sharper than 80° with 5-foot chords, the above value will be correct to the nearest foot. The accompanying tables are computed on this basis. See p. 39.

The expansion of the angle in terms of the tangent is, by Trigonometry,

$$t = \tan t - \tfrac{1}{3}\tan^3 t + \tfrac{1}{5}\tan^5 t - .$$

Substituting, and retaining nothing higher than k^5

$$d_0 = \frac{kl^2}{3} - .002\,823k^3l^6 - .000\,068k^5l^{10}$$

$$= \frac{I}{3}n^2 - .002\,823I^3n^6 - .000\,068I^5n^{10},$$

by using for k its value $I \div l_1^2$ from (8), and calling the ratio

$$l \div l_1 = n.$$

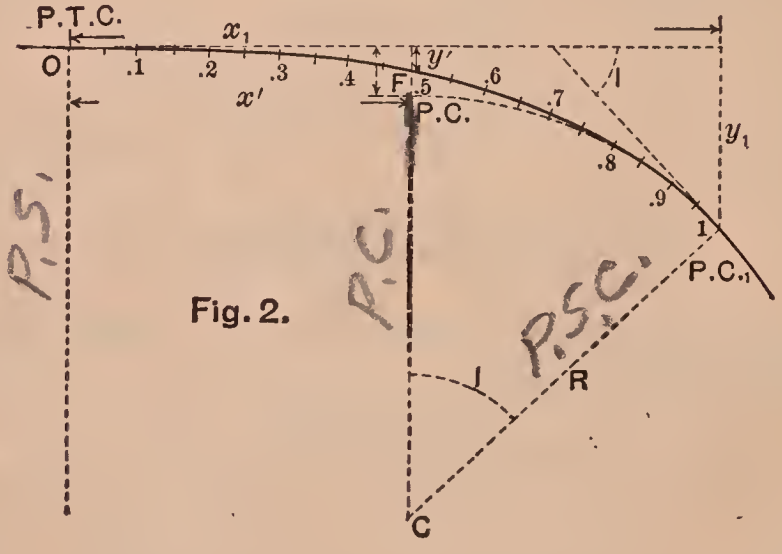

Fig. 2.

d_0 and I are in π-measure: to reduce to degree-measure, multiply by $\frac{\pi}{180} = .017\,453$, giving

$$d_0^\circ = \frac{I^\circ}{3}n^2 - .000\,023\,22\left(\frac{I^\circ}{3}\right)^3 n^6 - .000\,000\,001\,53\left(\frac{I^\circ}{3}\right)^5 n^{10};$$

$$d_0^\circ = \frac{I^\circ}{3}n^2 - B_0; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

i.e., *the deflection varies approximately as the square of the distance, and is nearly equal to one third the central angle subtending the same arc.*

B_0, in thousandths of a degree, can be taken from Table III with I° as argument.

With the instrument at an intermediate point, x'', y'',

$$\tan d'' = \frac{y - y''}{x - x''} = \frac{k}{3}(l^2 + l''^2 + ll'')$$
$$+ k^3[\tfrac{1}{105}(l^6 + l''^6) + \tfrac{3}{70}(l^5 l'' + ll''^5)$$
$$+ \tfrac{8}{105}(l^4 l''^2 + l^2 l''^4 + l^3 l''^3)] + k^5[.000\,167(l^{10} + l''^{10})$$
$$+ .002\,909(l^9 l'' + ll''^9) + .008\,985(l^8 l''^2 + l^2 l''^8)$$
$$+ .016\,604(l^7 l''^3 + l^3 l''^7) + .024\,223(l^6 l''^4 + l^4 l''^6)$$
$$+ .027\,556 l^5 l''^5].$$

From which, by finding the value of the angle in terms of the tangent, and substituting the value of k as before,

$$d''^\circ = \frac{I^\circ}{3l_1^2}(l^2 + l''^2 + ll'') - \text{Correction.}$$

$$d''^\circ = \frac{I^\circ}{3}(n^2 + n''^2 + nn'') - B'', \quad \ldots\ldots \quad (13)$$

where B'' contains the terms involving I^3 and I^5, a rapidly converging series.

With the instrument at Sta. 1 ($n'' = 1$),

$$d_1{}^\circ = \frac{I^\circ}{3}(n^2 + 1 + n) - \text{Correction}$$

$$= \frac{I^\circ}{3}A_1 - B_1. \quad \ldots\ldots\ldots\ldots \quad (14)$$

With the instrument at Sta. .5 ($n'' = \frac{1}{2}$),

$$d^\circ{}_{.5} = \frac{I^\circ}{3}(n^2 + \tfrac{1}{4} + \tfrac{1}{2}n) - \text{Correction}$$

$$= \frac{I^\circ}{3}A_{.5} - B_{.5}. \quad \ldots\ldots\ldots\ldots \quad (15)$$

Instrument at Sta. .25 ($n'' = \frac{1}{4}$),

$$d^\circ_{.25} = \frac{I^\circ}{3}(n^2 + \tfrac{1}{16} + \tfrac{1}{4}n) - \text{Correction}$$

$$= \frac{I^\circ}{3} A_{.25} - B_{.25} \quad \ldots\ldots\ldots\ldots \quad (16)$$

Instrument at Sta. .75 ($n'' = \frac{3}{4}$),

$$d^\circ_{.75} = \frac{I^\circ}{3}(n^2 + \tfrac{9}{16} + \tfrac{3}{4}n) - \text{Correction}$$

$$= \frac{I^\circ}{3} A_{.75} - B_{.75} \quad \ldots\ldots\ldots\ldots \quad (17)$$

A and B can be taken from Table III, with n'', n, and I° as arguments. B is in thousandths of a degree. Second differences should be used in interpolating for A when I° is large.

6. Construction of Tables.—These formulas are too complicated for convenient use, although all the series converge rapidly. Tables thus become necessary.

Table I.—The length of curve, l_1, for a given D is first assumed in values increasing by 20 feet from 40 to 700 ft.; I°, the central angle, can be found by (9); x_1, y_1, the coördinates of $P.C._1$, are found by (4) and (6) by the aid of Table II;* x', F, the coördinates of the $P.C.$ of the circular curve, are found by (11); l', the length of the transition curve from the $P.T.C.$ to the $P.C.$, is most conveniently found by assuming a value a little less than $[x' + (l_1 - x_1) \div 32]$ for l in (5), and noting the resulting value of x; a second assumption may sometimes be necessary to bring the value of l used in the higher terms to the nearest tenth, which will usually give $l' - x'$ with sufficient accuracy; $l' - x'$ is tabulated as e; ϕ', the central angle corresponding with l', will $= (l' \div l_1)^2 I$, by (2); when y', the ordinate opposite the $P.C.$, can be found by (4), or it can be found from (3) without using ϕ'; $l_1 - l'$ will give the length of the second portion of the transition curve; $100I^\circ \div D +$ arc excess, taken from Table V,† will give the

* They were actually computed by (3) and (5), so that occasionally there may be a slight discrepancy in the last place.

† Arc excess = excess of circular arc over the sum of the chords from the $P.C.$ to the $P.C._1$.

length of the circular arc for the corresponding portion; the difference between the transition arc and circular arc is tabulated as e'; $e + e' = \frac{l_1}{2} - x' -$ arc excess.

Table II gives C of (4) and (E) of (6), rendering the computation of the coördinates, x and y, of any point of the curve extremely simple.

Table III gives A and B for computing the deflection angles by (12), (14), (15), (16), (17), for 20 equidistant stations on the curve, with the instrument at each of 5 different ones; B is in thousandths of a degree, and can be interpolated for intermediate values of ϕ and I. A should be interpolated by second differences when I is large, or directly computed if more convenient. As great flexibility is thus given in the field-work in passing obstructions, etc., as with circular curves.

All the tabular quantities were computed to one or more places than have been retained, and they have been checked, in type, by second differences. It is confidently believed that no errors of importance will be discovered, and no inaccuracies which can affect the best grade of field-work.

Linear interpolation will usually be sufficient for intermediate values; second differences may be preferred for F and y in some parts of the table.

x_1, y_1, will usually be of use when an obstruction, or other local condition, prevents the use of the ordinary methods in the field.

Table IV.—For convenience, when the degree of the circular curve can be taken as a whole degree and the length of the transition curve can be slightly modified to suit the table, Table IV has been added giving the actual deflections in degrees and minutes required in running the curve. It is computed from Table III, but for fixed chord lengths rather than for a fixed number of chords, as being more convenient for tabulation and for use. The offsets F have been added from Table I, and the long chords from computation.

PRACTICE.

7. Field-work by Offsets.—It will usually be most convenient during location and construction to simply allow the offset F between the tangent and circular curve, and the

excess in length when important, without running the transition curve. Two hubs are then required at the *P.C.*, and also at the *P.T.*, one at the *P.C.* or *P.T.* proper, and the other on the tangent opposite, the distance between being *F*, one being set as a foresight and the other by offset. The instrument when brought up is placed over the offset hub, requiring the backsight to be similarly offsetted ; or, with a table of natural functions, the vernier may be set to

$$\cot^{-1}(\text{distance} \div \text{offset});$$

or, if the offset is small, at the angle in minutes

$$= \frac{\text{offset}}{.029 \times \text{distance in stations}}.$$

On a curve the offset should be multiplied by cos $(nD \div 2)$,* and the long chord taken for distance, in the above expression, when failing to offset the back flag.

The excess in length, $e+e'$, must be added to the station numbering, making the change for convenience at the *P.C.* and at the *P.T.* the same as the deficiency in length of the circle as compared with the sum of the tangents must be subtracted from the station numbering of the *P.T.* when the latter is set from the *P.I.* and the tangent continued without running the circular curve.

If intermediate points on the transition curve are needed during construction measure ordinates from the tangent proportional to the cubes of the distances from the *P.T.C.*, or from the circle proportional to the cubes of the distances from the $P.C._1$, *F* being about double the ordinate either way at the *P.C.*

Example 1. At station 674 a 12° curve to the left begins. Required the data for a 500′ transition curve (front tangent not fixed).

From Table I for $l_1 = 500$, $F = 21.60$, $e = .4$, $e' = 1.8$, $x' = 247.7$.

∴ at sta. 674, $-(e+e') = 673 + 97.8$, but called 674 to allow for excess in length, erect a perpendicular with transit or chain = 21.60; place hub; set transit over it with 0° of vernier parallel to tangent and run curve as usual. The *P.T.C.* will be $673 + 97.8 - x' = 671 + 50$, and the $P.C._1$, $676 + 50$.

* n = distance in stations; D = degree of curve.

$$\text{Sta. } 672, \quad \frac{21.60}{2} + 5^3 = .0864;$$

$$672 + 50, \quad .0864 \times 2^3 = .7;$$

$$673 \quad .0864 \times 3^3 = 2.3;$$

$$673 + 50, \quad .0864 \times 4^3 = 5.5;$$

and the same values for corresponding stations from the circle.

The true coördinates for any point can be computed by (4) and (6), if needed for greater accuracy. Thus, for sta. 673: Table I gives $I = 30°$, (2) gives $\phi° = (\frac{15}{60})^2\ 30^{\bullet} = 2°.7$. Table II gives $C = .0157$, $E = .00022$. $\therefore y = 150\ C = 2.36$; $x = 150 - 150E = 150$.

8. Field-work by Deflections.—For track centers the transition curve is best run with a transit. From F, I and l_1 are readily found from Table I, interpolating if necessary. Having l_1, the number of stations (an aliquot part of 20) can be taken so that they will be no farther apart than on the circular curve. The instrument points for running the curve are then decided upon, the corresponding numbers are taken from Table III to multiply by $I \div 3$ for the deflections, subtracting the corrections when important. The curve is then run in the usual way with transit and chain, but with short chords.

Blank pages are inserted after page 40 in order to preserve the deflections computed for the more common transition curves.

Regular stations can be used if preferred, it adding slightly to the labor of finding A in the deflection formula.

When the curve can be run with the instrument at the $P.T.C.$, —and $I°$ is not over 15° to 20°, the deflections can be taken by (12) equal $\frac{I}{3}n^2$, without the use of Table III.

When the degree of the curve and the offset can be taken from Table IV without interpolation, it will be found more convenient as the deflections can be taken out directly without computation.

Strictly, the distances measured should be along the arcs, and not along the chords; the chords can be shortened by Table VI to allow for this whenever the accuracy of the chaining will warrant; the maximum discrepancy is one inch per 100 feet, if the chords are as short as would be used for circular arcs of the same curvature.

In plotting, the radius of the circle is increased by the offset in fixing the center from the tangent, while the radius is used in describing the arc, thus leaving the offset F at the tangent point.

EXAMPLE 1.—Find the deflections for a 9° curve with a 10.43-ft. offset. Table I: $I=18°$; $I\div 3=6°$; $l_1=400'$; requiring 10 chords each 40'. With instrument at $P.T.C.$, from Table III:

$d_0{}^{.1}=.01\times 6°=0°\ 3'.6$; $d_0{}^{.8}=\ .64\times 6°\ =3°50'.4$;

$d_0{}^{.2}=.04\times 6\ =0\ 14\ .4$; $d_0{}^{.9}=\ .81\times 6\ -.003=4\ 51\ .4$;

. . . . . $d_0{}^{1}=1\ \times 6\ -.005=5\ 59\ .7$.

On reaching sta. 1 or the $P.C._1$, set up, backsight on sta. O, deflect $18°-d_0{}^1$ for tangent at sta. 1; then run the remainder of the circular curve as usual. On reaching the $P.T.C._1$, the second transition curve can be run continuously by placing the circle so that the vernier shall read $I°$ for the tangent at $P.T.C._1$, and zero when parallel with the tangent at the $P.T.$, and then setting the following readings computed from Table III.:

Instrument at $P.T.C._1$:

$d_1{}^{.9}=2.71\times 6°=16°\ 15'.6$; $d_1{}^{.2}=1.24\times 6°-.003=7°\ 26'.2$;

$d_1{}^{.8}=2.44\times 6\ =14\ \ 38\ .4$; $d_1{}^{.1}=1.11\times 6\ -.004=6\ \ 39\ .4$;

. . . . . $d_1{}^{0}=1\ \times 6\ -.005-5\ \ 59\ .7$.

EXAMPLE 2.—Find the deflections for a 9° curve with an 8.45 ft. offset. Table IV : 40-ft. chords; $I=16°\ 12'$; $l_1=360$, or 9 40-ft. chords. With instrument at $P.T.C.$, or sta. O:

Deflection for chord point				2 =	0°	16′
"	"	"	"	3 =	0	36
"	"	"	"	4 =	1	04
. . . . .						
"	"	"	"	9 =	5	23 .8
Tangent at		"	"	9 =	16	12

The deflections are also given for the instrument at chord point 2, 80 feet from the $P.T.C.$; chord point 4; chord point 6; chord point 8; and chord point 9, the $P.C._1$.

9. Best Length of Curve.—The rate of change of elevation of outer rail preferred by engineers, in passing from the tangent to the circular curve, varies from 1 to 4 inches per station; the elevation on the circular curve varies from one-

half an inch for low speeds, to 4 inches or more for very high speeds, per degree, until a maximum of from 5 to 9 inches is reached. This would require from 10 to 200 feet of transition curve per degree of main curve up to a length of some 600 feet.

On short tangents and short curves short transition curves will be required to prevent serious overlapping. On the other hand, a circular curve with transition ends leading to the tangents will usually approach nearer to the shape of a contour around a spur or up a narrow valley than a circular curve with tangents, thus using the longer curves in extending the combinations available for fitting the ground in rugged topography. This flexibility will also be found very convenient in running track centers over crooked portions of the line, as the line can be better fitted to the road-bed, and small discrepancies in re-running the curves from imperfectly recovered points can be taken care of in the offset, as any offset within the limits of the tables can be used by interpolation.

10. Compound Curves.—The general relation of a transition curve to a compound curve can be seen by reference to Table IV, which gives the curvature at each chord point of the transition curve and shows that the portion of the curve between any two chord points would form a transition curve connecting the two circular curves the same as the portion from the zero end or *P.T.C.* to any chord point forms a transition curve from the straight line to the curve.

To find the data required for locating the tangent points, as a check upon the transition curve, in Fig. 3, let *C* be the center of the sharper curve of radius *R* and degree *D*; *C'*, that of the flatter curve of radius *R'* and degree *D'*; F_2, the perpendicular distance *E'J'* between the curves on the line joining their centers; *OG''G*, the transition curve, joining the first curve at *G*, the second at *G''*, and its own tangent *OB* (not the tangent to the compound curve) at *O*; l_1, the length *OG*; *l''*, the length *OG''*; l_2, the length *G''G*; e_2, the excess in length by using the transition curve. Call *G* the $P.T.C._2$; *E'* or *J'*, the *P.C.C.*; *G''* the $P.C._2$; *E''* the *P.T.''*; and *O* the $P.T._2$.

Since the curvature increases directly with the distance,

$$\left.\begin{aligned} l'' &= l_1\frac{D''}{D}; \\ l_2 &= l_1 - l'' = l_1\frac{D - D''}{D}. \end{aligned}\right\} \quad \ldots\ldots \quad (18)$$

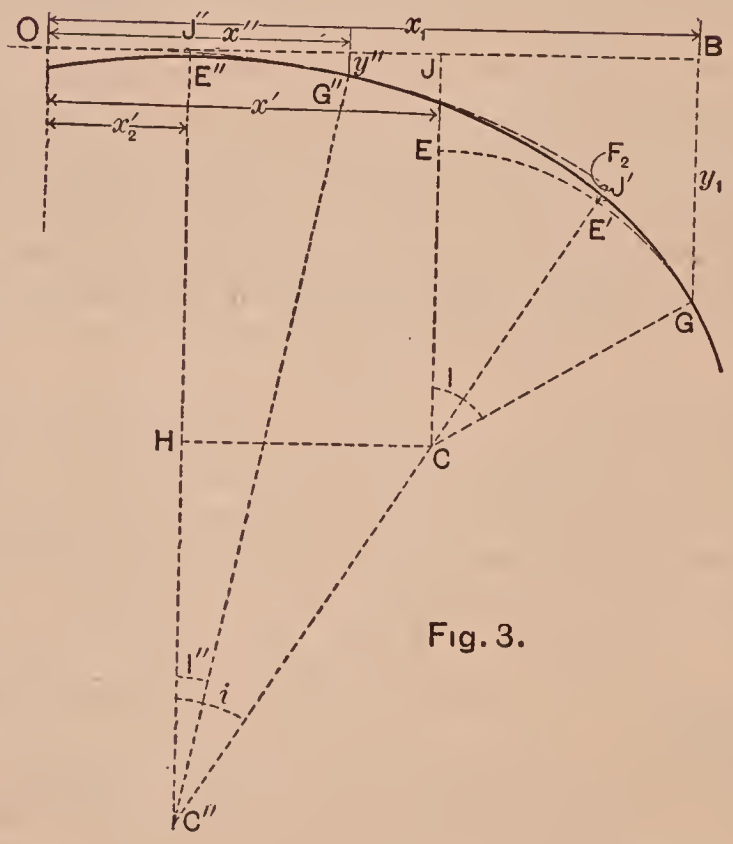

Fig. 3.

By equations (18) and (9),

$$I'' = \frac{l''D''}{200}, \quad = I\left(\frac{l''}{l_1}\right)^2 \quad \ldots\ldots \quad (19)$$

The coördinates of the center C are x' and $F + R$, taken from Table I for the $D°$ curve; those for C'' are x'_2 and $F'' + R''$, taken from Table I for the D'' curve with l'' as argument.

In the right triangle HCC'', HC, and HC'' are known from the above, giving

$$\left.\begin{aligned}
\tan i &= \frac{HC}{HC''} = \frac{x' - x'_2}{F'' + R'' - F - R};\\
CC'' &= \frac{HC''}{\cos i} = \sqrt{\overline{HC}^2 + \overline{HC''}^2};\\
F_2 &= R'' - R - CC'';\\
GE', \text{ or } P.T.C._2 \text{ to } P.C.C. &= \frac{I - i}{D}100;\\
J'G'', \text{ or } P.C.C. \text{ to } P.C._2 &= \frac{i - I''}{D''}100;\\
e_2 &= l_2 - (GE' + J'G'') - \text{arc excess};\\
&= l_2 - 100\left(\frac{I - i}{D} + \frac{i - I''}{D''}\right) - \text{arc excess};
\end{aligned}\right\} \quad \ldots \quad (20)$$

where "arc excess" is the sum taken from Table VI for the proper lengths for the two circular arcs.

Computing F_2 and e_2, for different cases, they are found to be the same as given by Table I, with $D - D''$ and l_2 as arguments, up to about $Dl_1 = 8000$; i.e., up to $D = 12°$, $l_1 = 680'$; $D = 20°$, $l_1 = 400'$; etc. For greater values, F_2 should be computed by (20) for accuracy, although the error is not large at the limits of the Table.

Computing GE', the distance from the $P.T.C._2$ to the $P.C.C.$ on the sharper curve, by (20), it is found equal to $l_2 \div 2$, up to about $Dl_1 = 4000$; leaving $J'G''$, the distance from the $P.C.C.$ to the $P.C._2$ on the flatter curve, $= l_2 \div 2 - e_2 -$ arc excess. For much larger values these distances should be computed by (20).

Hence, in using the offset method of allowing for the transition curve, assume a suitable l_2 (or F_2) for a $D - D''$ curve; take out the corresponding F (or l_1) and $e + e'$ from Table I; run the circular curve to E', the $P.C.C.$, as usual (the front curve not being closely fixed in position); offset radially an amount F_2; set up at J' and continue as usual, allowing for $e + e'$ when important.

In using the deflection method, assume the length l_2 between the two branches from Table IV, and take out the length of the transition curve to each point, l_1 and l''. From Table I take out the offset F_2 and the excess length e_2 ($= e + e'$) for a $D - D''$ curve. Then GE', the distance from the $P.C.C.$ to the tangent point of the sharper curve $= l_2 \div 2$; $J'G''$, the distance to the tangent point of the flatter curve, $= l_2 \div 2 - e_2 -$ arc excess for both circular curves taken from Table VI. Lay off GE' and $J'G''$, locating the tangent points

$P.T.C._2$ and $P.C._2$; then set up at either one and run the curve from the deflections given in Table IV.

If Dl_1 exceeds the limits given above, F_2, e_2, GE' and $J'G''$ must be computed by (20) as already indicated.

If the length of the transition curve cannot be assumed from Table IV, it can be assumed to correspond with Table I, and F_2 and e_2 taken out as above. The deflections will then have to be computed from Table III, making it more convenient to run the curve from the $P.T.C._2$, the tangent point with the sharp curve, unless the $P.C._2$ should happen to come at an instrument point of the table.

EXAMPLE 1.—A 10° curve compounds to a 4° with a 3.92 ft. offset, at sta. 196 + 40. Find the data for running the transition curve with a transit.

Table I, for $F_2 = 3.92$, and $D - D'' = 6°$, gives $l_2 = 300$ ft.

By (18), $l_1 = l_2 \frac{D}{D - D''} = 500$ ft.; $l'' = l_1 - l_2 = 200$ ft.

Table I, for $D = 10°$, $l_1 = 500$, gives $I = 25°$, $F' = 18.05$, $x' = 248.4$; for $D'' = 4°$, $l'' = 200$, gives $I'' = 4°$, $F'' = 1.16$, $x'_2 = 100$.

Table V gives $R = 573$, $R'' = 1432.5$.

$Dl_1 > 4000$, requiring a computation for GE' by (20) for strict accuracy.

$$\tan i = \frac{248.4 - 100}{1.16 + 1432.5 - 18.05 - 573} = \frac{148.4}{842.61},$$

giving $i = 9°.989$.

$$GE' = \frac{25 - 9.989}{10} 100 = 150.11 \text{ ft.}$$

$$J'G'' = \frac{9.989 - 4}{4} 100 = 149.72 \text{ ft.}$$

$$P.T.C._2 = \text{sta. } 196 + 40 \quad - (1 + 50.1) = 194 + 89.9;$$

$$P.C._2 \quad = \text{“ } 194 + 89.9 + (3 + 00) \quad = 197 + 89.9.$$

Table IV, instrument at $P.T.C._2$, or chord point 10.

Tangent at	$C.P.$	10 =	25° 00′
Deflection for	“	9 =	22 35
“	“	8 =	20 20
“	“	4 =	12 59.8
Tangent at	“	4 =	4 00

Zero at $P.T.C._2$ makes an angle = 25° with tangent to 10° curve.

Tangent at $P.C._2$ makes with backsight an angle = 12° 59′.8 − 4° = 8° 59′.8.

In this example the deflections could have been taken from Table III. with nearly the same readiness.

11. Tangent Distance with Transition Curve.—Given the central angle Δ, the degree of the curve D, and the offset F, to find the tangent distance T'.

The effect of the transition curve is to move the circular curve from the position *HLM* to *ENE'*, the perpendicular dis-

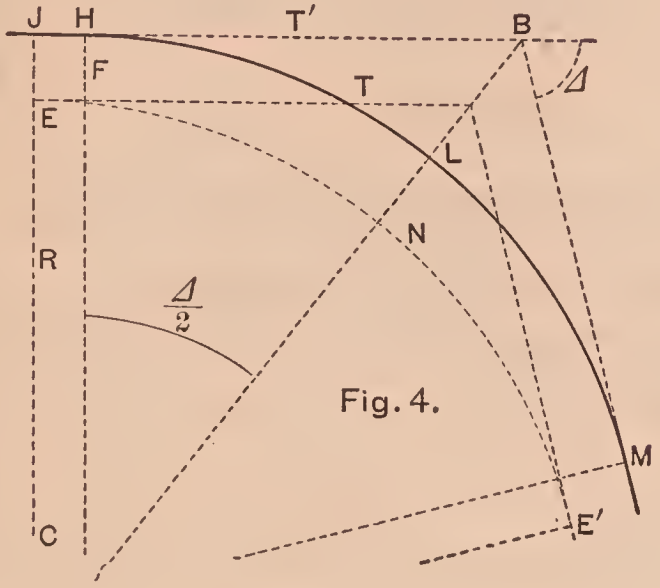

Fig. 4.

tance between the tangents being *F*. In the right triangle *JCB*,

$$JB = (EC + JE) \tan \frac{\Delta}{2},$$

or

$$T' = (R + F) \tan \frac{\Delta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$= T + F \tan \frac{\Delta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$T_1 = R \tan \frac{\Delta}{2}$, being the tangent distance without the transition curve, found by Table VII, or by computation.

If the curve is to be run by deflections, x' must be added to T' to reach back to the *P.T.C.*, giving

$$T_1 = T + F \tan \frac{\Delta}{2} + x' \quad . \quad . \quad . \quad . \quad (23)$$

x' can be taken from Table I with D and F as arguments.

EXAMPLE 1.—Write out the transit notes for a 6° curve, with *P.I.*, point of intersection of the two tangents, = sta. 721 + 52.7; $\Delta = 17° 21'$; $F =$ 2.51 ft.

By (21), $T' = (955 + 2.51) \tan 8° 40\frac{1}{2}' = 146.1$.

Length of curve $= \frac{\Delta}{D} = 2.892$ stas.

TRANSIT NOTES.

Sta.	Point.	Curve or Bearing.	Vernier.
+ 95.8	*P.T.*, $F = 2.51$	S. 35° 30′ W.	8° 40½′
2			5 48
1			2 48
+ 6.6	*P.C.*, $F = 2.51$	6° R.	
720		S. 18° W.	

For track centers, or transition curve run with transit:
By (23) and Table I, $T_1 = 146.1 + 119.9 = 266$. $I° = 7°.2$.

Length of circular curve $= \frac{\Delta - 2I}{D} = 0.492$ stas.

Length of transition curve, Table I, = 240 ft., requiring 4 60-ft. chords.
Table III, for $I = 7°.2$:
Instrument at sta. O: $d_0^{.25} = 0° 09'$; $d_0^{.5} = 0° 36'$; $d_0^{.75} = 1° 21'$; $d_0^{1} = 2° 24'$.

Instrument at sta. 1: $d_1^{1} = 7° 12'$; $d_1^{.75} = 5° 33'$; $d_1^{.5} = 4° 12'$; $d_1^{.25} = 3° 09'$; $d_1^{0} = 2° 24'$.

Table IV, 40-ft. chords would give the deflections also, but for 6 chords instead of 4.

TRANSIT NOTES.

Sta.	Point.	Curve or Bearing.	Vernier.	Remarks.
724 + 15.9⊙	*P.T.*$_1$	S. 35° 30′ W.	2° 24′	Vernier = 2° 24′ for B. S.
.25			3 09	
.5			4 12	
.75			5 33	
+ 75.9⊙	*P.T.C.*$_1$		1 28½	Vernier = 8° 40½′ for B. S. (= $I°$ + 1° 28½′)
721 + 26.7⊙	*P.C.*$_1$	6° R.	2 24	Vernier = 4° 48′ for B. S. (= $I°$ − 2° 24′)
.75			1 21	(These settings for B. S.
.5			0 36	give 0° on the tang. for
.25			0 09	P.C.$_1$ and P.T.$_1$, and $I°$ on
+ 86.7⊙	*P.T.C.*			the tang. for P.T.C.$_1$.)
718		S. 18° W.		

12. Tangent Distance with Unequal Offsets.— From Fig. 5,

$$\left.\begin{aligned} T' &= T + AH - BJ \\ &= T + \frac{F''}{\sin \Delta} - \frac{F'}{\tan \Delta} \\ &= R \tan \frac{\Delta}{2} + \frac{F''}{\sin \Delta} - \frac{F'}{\tan \Delta}. \end{aligned}\right\} \quad . \quad . \quad . \quad (24)$$

$$\left.\begin{aligned} T'' &= T + BH - GH \\ &= T + \frac{F'}{\sin \Delta} - \frac{F''}{\tan \Delta} \\ &= R \tan \frac{\Delta}{2} + \frac{F'}{\sin \Delta} - \frac{F''}{\tan \Delta} \end{aligned}\right\} \quad . \quad . \quad . \quad (25)$$

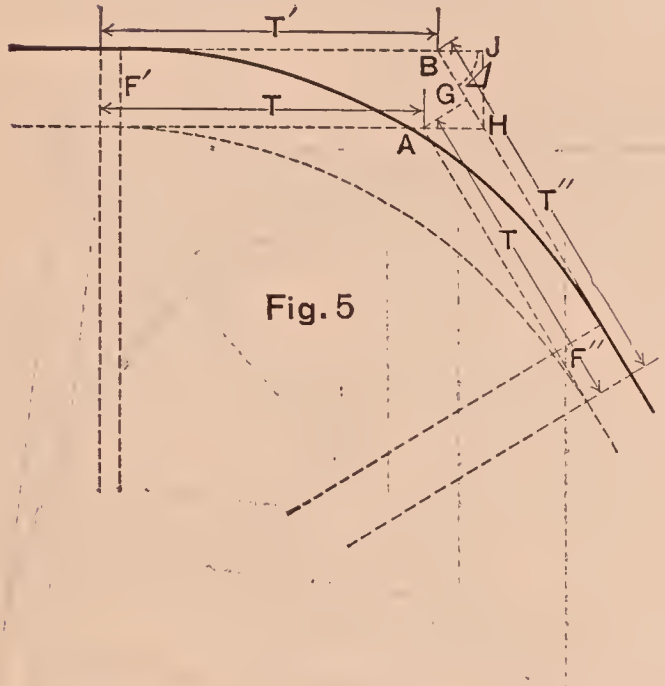

Fig. 5

Example 1. Compute the data for a 4° curve; $P.I.$ = sta. 54 + 35.6; $\Delta = 23° 15'$; $F' = 2.62$ ft.; $F'' = 0.94$ ft.

By (24), $$T' = 294.7 + \frac{0.94}{.3947} - \frac{2.62}{.4296} = 291.0.$$

(25), $$T'' = 294.7 + \frac{2.62}{.3947} - \frac{0.94}{.4296} = 299.2.$$

$$\text{Length of curve} = \frac{\Delta}{D} = 5\ 812 \text{ stas.}$$

$\therefore P.C. = 51 + 44.6$, with offset $F' = 2.62$

$P.T. = 57 + 25.8$, with offset $F'' = 0.94$.

No further attention need be given the transition curves, except to allow the proper offsets until track centers are required.

Ex. 2. Write out the transit notes for Ex. 1 for track centers.

13. Transition Curve added without changing the Position of the Line opposite Vertex.— Let E be the external for the old curve, and E' that for the new. In Fig. 6,

$$PN = BN - BP.$$

$$\therefore E' = E - F \sec \frac{\Delta}{2}. \quad . \quad . \quad . \quad . \quad . \quad (26)$$

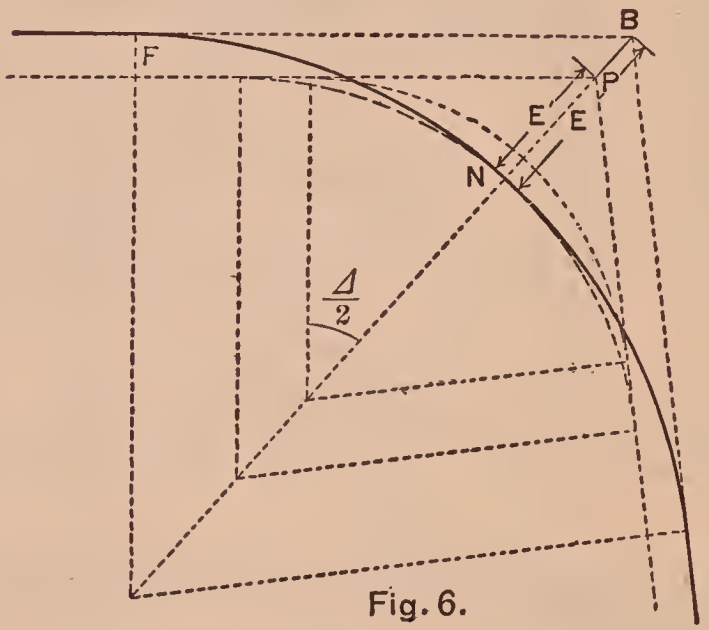

Fig. 6.

Divide the external distance for a 1° curve, Table VII, by D, for E, or compute it from $E = R\left(\sec \frac{\Delta}{2} - 1\right)$. Then find E' by (26), and divide the external for a 1° curve by it for D', the degree of the curve required.

D' can always be changed to the nearest whole minute for convenience in running without sensibly disturbing the required position.

It will be larger than D.

EXAMPLE 1. A 4° curve beginning at sta. 80 and ending at sta. 85 is to be replaced by a curve with a 1-ft. offset passing through the same point opposite the vertex.

Required the field-notes running the entire curves with a transit and using regular stas.

Table VII : for $\Delta = 20°$, external for 1° curve = 88.40, tangent = 1010.4.

$$\therefore E = 22.10,\ T = 252.6.$$

$$E' = 22.10 - \frac{1}{.985} = 21.08;$$

$$D' = \frac{88.40}{21.08} = 4°\ 11'.6, \text{ say } 4°\ 12';$$

$$T' = \frac{1010.4}{4.2} + .2 = 240.8.$$

Table I :

$$\text{for } l_1 = 180,\ D = 4°\ 12';\ F = .94 + \tfrac{12}{60}(1.18 - .94) = 0.99;$$

$$l_1 = 200,\ D = 4°\ 12';\ F = 1.16 + \tfrac{12}{60}(1.45 - 1.16) = 1.22.$$

$$\therefore \text{for } F = 1,\ D = 4°\ 12';\ l_1 = 180 + 20\frac{1 - 0.99}{1.22 - 0.99} = 180\ 9,\ x' = 90.4.$$

By (9), $I° = \frac{l_1 D}{200} = 3°.799.$

$$\text{Length of circular curve} = \frac{20 - 2 \times 3.799}{4.2} = 2\ 953 \text{ stas.}$$

$$P.T.C. = \text{sta } 80 + T - T' - x' = 79 + 21\ 4.$$

The d's are interpolated from Table III.

TRANSIT NOTES.

Sta	Point.	Vernier.	Remarks.
+ 78.5⊙	$P.T._1$	1° 16′	Vernier = 1° 16′ for backsight.
5		2 3.3	
4			
+ 97 6⊙	$P.T.C._1$	6 12	Vernier = 10° for backsight
3		4 9	(= $I°$ + 6° 12′).
2		2 3	
+ 2.3⊙	$P.C._1$	1 16	Vernier = 2° 32′ for backsight
1		1 13.8	(= $I°$ − 1° 16′).
80		0 14.4	
79 + 21.4⊙	$P.T.C.$		

14. Transition Curve added with the Least Deviation from the Old Roadbed, in improving Old Track.—For this case the new track should pass about as far outside opposite the vertex as it passes inside opposite the $P.C.$, or about $F \div 2$.

$$\therefore E' = E - F \sec \frac{\Delta}{2} - \frac{F}{2}; \quad . \quad . \quad . \quad . \quad (27)$$

the problem otherwise being the same as § 13. Instead of $F \div 2$, any other value can be used as desired.

15. Transition Curve added without changing the Length of the Track, to avoid cutting the Rails.—In Fig. 7, let A and H be the $P.C.$ and $P.T.$ for

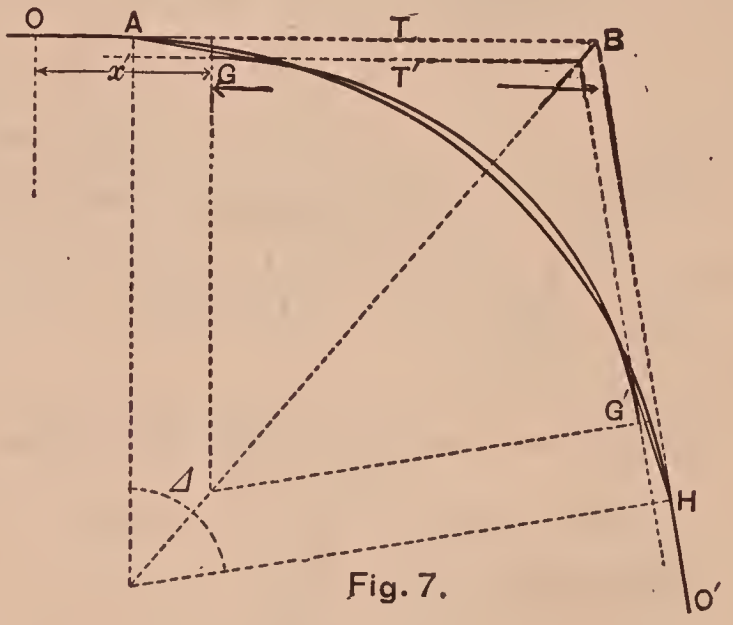

Fig. 7.

the old track; G and G' the $P.C.$ and $P.T.$, O and O' the $P.T.C.$ and $P.T._1$, for the new track.

By the old line,

$$\begin{aligned}\text{Dist. from } O \text{ to } O' &= OA + \text{arc } AH + HO' \\ &= 2OA + R\Delta \\ &= 2(T'' + x' - T) + R\Delta \\ &= 2\left((R' + F)\tan\frac{\Delta}{2} + x' - R\tan\frac{\Delta}{2}\right) + R\Delta;\end{aligned}$$

where Δ is in π-measure $= .017453\Delta°$.

By the new line,

$$\begin{aligned}\text{Dist. from } O \text{ to } O' &= 2(x' + e + e') + \text{arc } GG' \\ &= 2(x' + e + e') + R'\Delta;\end{aligned}$$

where x' and $e + e'$ are taken from Table I.

Placing the two values equal,

$$R'\Delta + 2(x' + e + e') = 2\left((R' + F)\tan\frac{\Delta}{2} + x' - R\tan\frac{\Delta}{2}\right) + R\Delta;$$

$$R' = \frac{.01745R\Delta^\circ - 2(R - F)\tan\frac{\Delta}{2} - 2(e + e')}{.01745\Delta^\circ - 2\tan\frac{\Delta}{2}}; \quad . \quad . \quad . \quad (28)$$

or

$$F = \frac{.008725(R' - R)\Delta^\circ + (e + e') + (R - R')\tan\frac{\Delta}{2}}{\tan\frac{\Delta}{2}}. \quad (29)$$

This problem is usually limited to small values of F or to small changes in R, so that $e + e'$ is practically known. For larger changes a second trial may be necessary.

EXAMPLE 1. Track has been laid on a 5° curve 900′ long. Required the data for transition curves without changing the length of the rails. Assume $F = 1.5$ ft., as it will give by § 9 an easy transition without too great shift of track.

Table I; $e + e' = 0$. Also, $\Delta = 45°$; $R = 5730 \div 5 = 1146$.

$$(28),\ R' = \frac{899.90 - 948.13 - .00}{.78525 - .82842} = 1117 \text{ ft.}$$

Changing R' to 1116.2 ft.,

$$D' = 5°\ 08'; \quad L' = 45 \div 5.133 = 8.767 \text{ stas.}; \quad T' = (1116.2 + 1.5)\tan\frac{\Delta}{2} = 463.0;$$

T for old curve = 474.7, requiring the $P.C.$ to be moved forward 11.7 ft.

Table I; by interpolation $l_1 = 200.7$ ft.

With stakes 100′ apart up to 8° curves, it is unnecessary to run the transition curve with an instrument; $y'(= F \div 2$ in this case) locating a point opposite $P.C.$; x' measured back on the tangent locating $P.T.C.$, and I, $= l_1 D \div 200$, locating the $P.C._1$, each being distant $l_1 \div 2$ from the $P.C.$ in this case.

16. Transition Curve added without disturbing the Central Portion of the Circular Curve.—To make room for a given offset F, the end of the circular curve must be sharpened by compounding, and to prevent too great a difference in curvature at the $P.C.C.$, the change should be limited to, say, 1° to 2°, i.e., $D'^\circ - D^\circ < 2°$.

The two circular curves are tangent to each other at G, Fig. 8, and the angle between them increases directly with the distance. Hence the offset from one to the other must increase with the square of the distance so long as the angle can be taken proportional to its sine, and the distance can be taken the same by either curve.

At 100 feet from G,

$$\text{Offset} = 100 \sin \tfrac{1}{2}(D' - D) = 50 \sin 1^\circ\,(D'^\circ - D^\circ)$$
$$= \tfrac{7}{8}(D'^\circ - D^\circ).$$

Hence at n stations from G,

$$\left.\begin{aligned} &\text{Offset} = \tfrac{7}{8}n^2(D'^\circ - D^\circ),^* \text{ approximately, or,} \\ &F < \tfrac{7}{4}n^2, \text{ for } (D'^\circ - D^\circ) < 2^\circ. \end{aligned}\right\} \quad . \quad . \quad (30)$$

From this approximate value of n assume a convenient Δ_1. Then in Fig. 8,

$$HE = AC - AH - CE,$$

or
$$R' = R - F - (R - R')\cos \Delta_1,$$

from which

$$R' = R - \frac{F}{1 - \cos \Delta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Having R', the transition curve can be put in as usual.

For flat curves and short transition curves the problem can be simplified as follows:

Assume the $P.C._1$ from 100 to 200 feet from the $P.C.$; measure the perpendicular distance y_1 to the tangent, and the angle I which the curve at the $P.C._1$ makes with the tangent. If the curve is regular and the $P.C.$ can be accurately found, y_1 and I can be computed from AG or n. The coefficients C and E of equations (4) and (6) can be found from Table II with I as argument, giving

$$l_1 = \frac{y_1}{C}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$x_1 = l_1 - l_1 E. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

---

* This formula is very convenient in location, in computing the change of curvature required for a given offset. When $D = 0$, it reduces to

$$F = \tfrac{7}{8}n^2 D,$$

the offset from a tangent to a circular curve.

Measure x_1 back on the tangent from the foot of the perpendicular at $P.C._1$ for the $P.T.C.$ One or two intermediate points can be located by coördinates computed by Table II; or the curve can be run with transit, computing the deflection angles by Table III, or taking them proportional to the square of the distance with the instrument at the $P.T.C.$, the last one being $I \div 3$, which will give a check on the work.

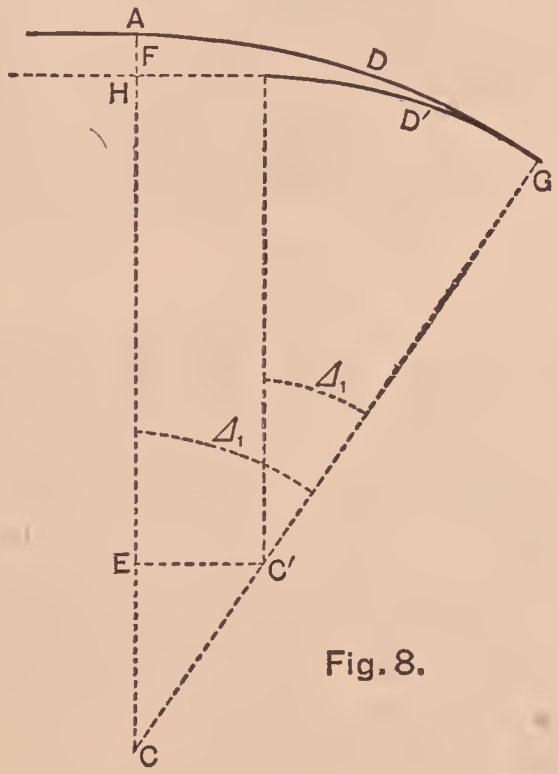

Fig. 8.

An approximate value of l_1 can be found from (32) by substituting the first term of the equation above (4) for C, giving

$$l_1 = \frac{3y_1}{.017453I^\circ}.$$

By the footnote to (30),

$$y_1 = \tfrac{7}{8}n^2D.$$

Substituting,

$$l_1 = 150n^2 \frac{D}{I^\circ},$$

or

$$l_1 = 150n; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

i.e., *the length of the transition curve is once and a half that of the circular curve from the P.C. to the P.C.*$_1$.

The degree of curvature at the *P.C.*$_1$ as found by (9) will be

$$D' = \frac{200I^\circ}{l_1};$$

by (34),

$$= \frac{200I^\circ}{150n} = \frac{200}{150}D,$$

or

$$D' = \tfrac{4}{3}D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The method is thus admissible to 3° curves if the break in curvature is limited to 1°, to 6° curves if the break is limited to 2°, etc., independently of the distance from the *P.C.* to the *P.C.*$_1$, or of the amount of the old track disturbed.

It can be extended to the case where there is an offset between the tangent and the circular curve at the *P.C.*, if desired; although it then offers no advantages over the regular methods already given, while it has the disadvantage of usually introducing a break in curvature at the *P.C.*$_1$. As the offset at the *P.C.* increases relatively to the offset y_1 at the *P.C.*$_1$, the break in curvature decreases, reaching zero when the ratio becomes 1 : 4; the transition curve having the flatter curvature ($D' < D$) at the *P.C.*$_1$ for all greater ratios.

Example 1. Find the data for a 4-ft. offset for a long 12° curve without disturbing the central portion.

(30), $4 < \frac{7}{4}n^2$, giving $n > 1.51$.

Assume $\Delta_1 = 24°$, giving

$$(31), \quad R' = 477.5 - \frac{4}{1 - .91355} = 431.23.$$

Table V, $D' = 13° 17'$.

The *P.C.* will be moved forward, $= 200 - \frac{24}{D'}100 = 19.3$ ft.

Table I, with $F = 4$ ft., $D' = 13° 17'$; $l_1 = 203.7$; $x' = 101.6$; $I = \frac{l_1 D'}{200} = 13°.529$; $P.C.$ to $P.C._1 = l_1 \div 2 = 101.8$.

The deflections can now be found from Table III, or the x and y coördinates from Table II.

By the second method; assume the $P.C._1$, say 100 feet on the curve; the $P.T.C.$ by (34) will be 50 feet back on the tangent; and the middle ordinate will be $\frac{1}{8}$ that at $P.C._1$; the latter being readily measured on the ground, or it can be computed. The break in curvature at the $P.C._1$, however, will be by (35) $D' - D = \frac{4}{3} \times 12 - 12 = 4°$, as compared with 1° 17′ at the $P.C.C.$ by the first method. The former cannot be reduced, while the latter can be by diminishing F, or by increasing the distance from the $P.C.$ to the $P.C.C.$

17. Transition Curve added at the *P.C.C.* of a Compound Curve by changing the Degree of the Second Branch.—Draw a tangent $AB = T$ at the $P.C.C.$, Fig. 9, to meet the tangent at the $P.T.$ at B, and measure AB and Δ.

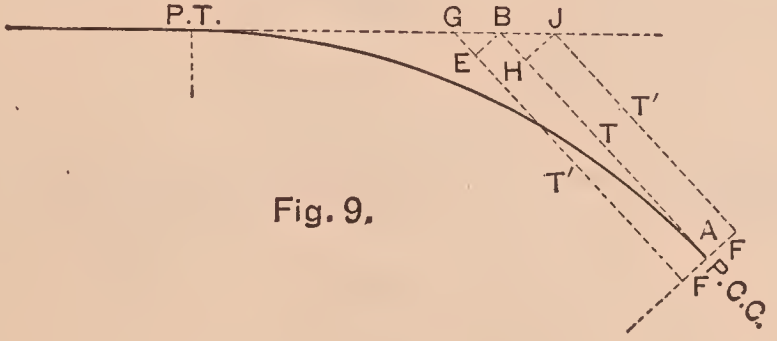

Fig. 9.

Let the required tangent be T'.

In the right triangles BJH and BEG,

$$GE = BH = F \cot \Delta.$$

$$\therefore T' = T \pm F \cot \Delta; \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where the plus sign is for the second branch sharper than the first, and the minus sign for the second branch flatter than the first.

To find D', divide the tangent for a 1° curve, Table VII, by T', or

$$R' = T' \cot \frac{\Delta}{2}, \quad D' = \frac{5730}{R'}.$$

Having D', proceed as usual.

If an offset F' is added at the $P.T.$ also, T' will be shortened by $F' \div \sin \Delta$, giving

$$T'' = T \pm F \cot \Delta - \frac{F'}{\sin \Delta}, \quad \ldots \quad (37)$$

with which proceed as above.

An offset is readily added at the $P.C.$ if desired, from what has already been given.

EXAMPLE 1. A 20° curve compounds to a 12° at sta. 844, with the $P.T.$ at sta. 850.

Required the data for a 3-ft. offset at the $P.C.C.$ and a 7-ft. offset at the $P.T.$

$$\Delta = 72°; \; F = 3; \; F' = 7; \; T = R \tan \frac{\Delta}{2} = 346.9.$$

By (37), $T'' = 346.9 - 3 \times .325 - \frac{7}{.951} = 338.5.$

$$D'' = \frac{5730}{338.5 \times 1.37638} = 12°.3 = 12° \; 18'.$$

The new vertex comes opposite a point on the tangent back of the old by an amount

$$= \frac{F}{\sin \Delta} + F' \cot \Delta$$

$$= \frac{3}{.951} + 7 \times .325 = 5.4 \text{ ft.},$$

which places the new $P.T.$ back of the old an amount

$$= 5.4 + (346.9 - 338.5) = 13.8 \text{ ft.}$$

18. Transition Curve added at the *P.C.C.* of a Compound Curve by moving the *P.C.C.*

Let R_1 = the radius and C the center of the first branch; R_2 = the radius and C_1 the center of the second branch; Δ_2 = the central angle of the second branch; Δ'_2 = the required central angle with the offset F added at the $P.C.C.$; Δ''_2 = the required central angle with the offset F added at the $P.C.C.$ and F_2 at the $P.T.$

(*a*) $R_2 < R_1$ (Fig. 10).

With C as center and $R_1 - R_2 - F$ as radius, describe the arc GG' to meet a parallel to the tangent at $P.T.$ drawn

through the center C_1. G will be the new center required, and J the new $P.C.C.$

In the right triangle CC_1K,

$$CK = CC_1 \cos \Delta_2 = (R_1 - R_2) \cos \Delta_2.$$

In the right triangle CGK,

$$\cos \Delta'_2 = \frac{CK}{CG};$$

$$\cos \Delta'_2 = \frac{(R_1 - R_2) \cos \Delta_2}{R_1 - R_2 - F} \quad \ldots\ldots \quad (38)$$

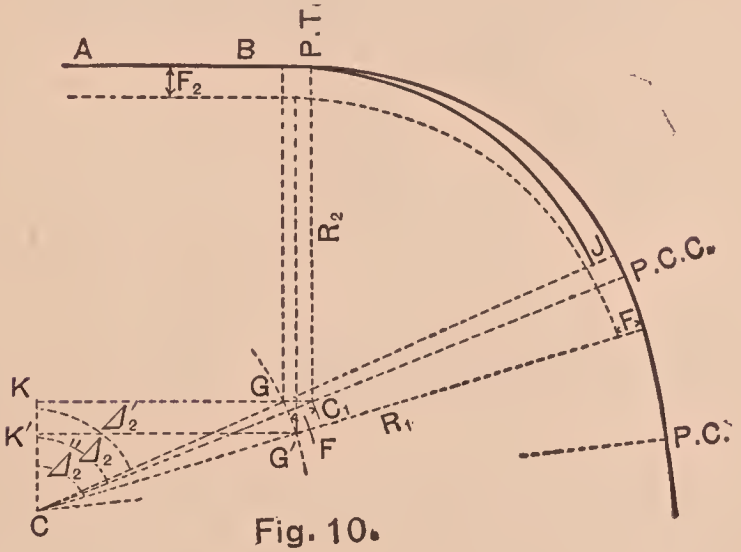

Fig. 10.

With an offset F_2 at the $P.T.$, G' will be F_2 farther from AB than G, giving

$$\cos \Delta''_2 = \frac{CK'}{CG'};$$

$$\cos \Delta''_2 = \frac{(R_1 - R_2) \cos \Delta_2 - F_2}{R_1 - R_2 - F} \quad \ldots\ldots \quad (39)$$

(*b*). $R_2 > R_1$. (Fig. 11.)

$$CK = (R_2 - R_1) \cos \Delta_2;$$

$$\cos \Delta'_2 = \frac{(R_2 - R_1) \cos \Delta_2}{R_2 - R_1 - F} \quad \ldots\ldots \quad (40)$$

Or, with offset F_2 at $P.T.$,

$$\cos \Delta_2'' = \frac{(R_2 - R_1)\cos \Delta_2 + F_2}{R_2 - R_1 - F}. \quad . \quad . \quad . \quad (41)$$

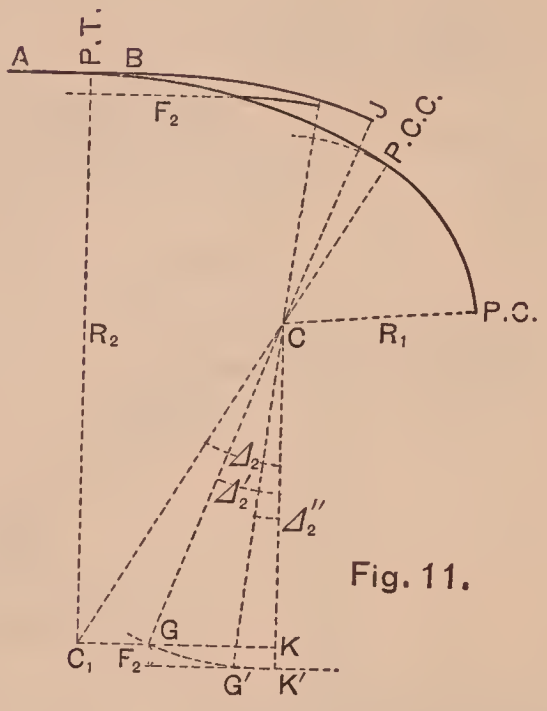

Fig. 11.

Example 1. A 6° curve begins at sta. 672, compounds to an 18° at sta. 680, with a tangent at sta. 685. Required the data for an offset of 5.01 ft. at the $P.C.C.$, and 32 ft. at the $P.T.$

$$\Delta_2 = 90°;\ F = 5\ 01;\ F_2 = 32 \text{ ft.}$$

$$(39), \quad \cos \Delta''_2 = \frac{(955 - 318.33)0 - 32}{955 - 318.33 - 5.01};$$

giving $\Delta''_2 = 92° 54'$.

$$\therefore P.C.C. = 680 + \frac{\Delta_2 - \Delta''_2}{D_1} = 679 + 51.7;$$

$$P.T. = 679 + 51.7 + \frac{\Delta''_2}{D_2} + (e + e') = 684 + 68.0;$$

taking $e + e'$ from Table I, for $D = 12°$, $F = 5.01$.

The deflections for the transition curve with the instrument at the $P.T.C.$ would be computed as follows, using the notation of § 10 and Fig. 3.

Table I, for $D - D'' = 12°$, and $F = 5.01$, gives $l_2 = 240$.

$l_1 = 240 \times \frac{18}{12} = 360$, requiring 10 36-ft. chords; $l'' = 360 - 240 = 120$.

$D = 18°$, $l_1 = 360$, gives $I = 32°.4$, $x' = 178.1$, $F + R = 16.76 + 318.33 = 335.09$.

$D'' = 6°$, $l'' = 120$, gives $I'' = 3°.6$, $x'' = 60$, $F'' + R'' = .63 + 955 = 955.63$.

(20), $$\tan i = \frac{178.1 - 60}{955.63 - 335.09}; \quad i = 10° \; 46'\tfrac{1}{2};$$

$$G''J' = P.C._2 \text{ to } P.C.C. = \frac{i - I''}{D''} 100 = 119.6;$$

$$E''G'' = P.T.'' \text{ to } P.C._2 = \frac{I''}{D''} 100 = 60;$$

$$E'G = P.C.C. \text{ to } P.T.C._2 = \frac{I - i}{D} 100 = 120.1.$$

Table III.

$d_0{}^{.1} = .01 \times 10°.8 = 0° \; 6\tfrac{1}{2}'$;

. . . . . .

$d_0{}^{\frac{1}{3}} = \frac{1}{3} \times 10.8 = 1 \; 12 = P.C._2$;

. . . . . .

$d_0{}^1 = 1 \times 10.8 - .030 = 10 \; 46 = P.T.C._2$.

$\therefore$ set up at E'' at sta. $679 + 51.7 - 119.6 - 60 = 677 + 72.1$, turn tangent; measure $x'' = 60$ back; offset $F'' = .63$, giving O; set up at O and run transition curve with above deflections and 36-ft. chords, except when interpolating $d_0{}^{\frac{1}{3}}$.

Table IV, 20-ft. chords would give the deflections without computation, and allow of setting up at the $P.C._2$ as well as at the $P.T.C._2$.

19. Old Track.—The problems already given will suffice for all the ordinary cases in aligning old track, where the curves are short and the tangents can be run to intersect at the *P.I.*, or can be connected across by a single tie-line.

With long sharp curves, where the roadbed usually lies in a deep cut or high fill, so that it is inconvenient to prolong the tangents to intersect, other methods will be found more desirable. On account of the trackman's habit of springing the tangent points in, making a sort of transition curve in his effort to secure an easy-riding track, the degree of curve can best be found by setting up on the central portion, and noting the deflections per station each way from the instrument. A value can then be assumed, and a trial-line extended through in each direction, noting distances from the center of the track

at different points, and especially at the ends, in order to determine the offsets to the tangents.

The change in deflection, when small, required to pass the line through any given point can be found from (30),

$$\text{Offset} = \tfrac{7}{8}n^2(D' - D) = \tfrac{7}{4}n^2(d' - d);$$ *

where n = distance in stations, D and d the trial degree of curve and deflection, respectively, and D' and d' the required ones.

The change in the position of the first instrument point with reference to the center line, and the deflections which will best fit the roadbed, can thus be readily found. Slight changes in deflection, starting at instrument points, are admissible in order to lessen the shift of the track, the effect being to slightly compound the curve; while any slight discrepancy in the work can be thrown into the offset at the tangent and taken care of in the transition curve.

If in thus fitting the roadbed a suitable offset cannot be secured for the transition curve without too great expense for track-work, the end may be compounded by one of the methods of § 16. If the track is shifted more than, say, 2 feet, the estimated cost of track-work should include the extra cost of maintenance, due to the settlement of the ends of the ties on the new roadbed.

If the original curve is compound, the *P.C.C.* will be difficult to find directly with accuracy, owing to the approximate transition curve introduced by the trackman. In running the trial-line which fits the first branch of the compound curve, the distances from the center of the track will begin to increase in passing onto the second branch. Measuring two of these and substituting in (30),

$$\text{Offset} = \tfrac{7}{8}n^2(D' - D);$$
$$\text{Offset}' = \tfrac{7}{8}(n + 1)^2(D' - D);$$

from which to find n and D', the other quantities being known. Or, D' can be found directly in the same manner as D, when one measured offset will give n, or the distance from the offset to the *P.C.C.*

Having the *P.C.C.*, if the change in curvature is great a transition curve can be added by the methods given in §§ 17, 18. Or a short transition curve can be added without chang-

* Searles's *Field Engineering*, Table X and §§ 146-152, on the valvoid curve, will give more accurate results, but with more labor.

ing the main portion of either branch of the compound curve by sharpening the sharp curve, and flattening the flat one by say 1°, for a distance n from the *P.C.C.*, so that the two changes will give the desired offset by (30). The curvature on one side will thus be increased as much as it is flattened on the other, leaving the two branches parallel at the *P.C.C.*, and at the offset distance apart. The transition curve can then be run by § 10; or, with a close approximation for short transition curves, by taking the ordinates starting from the $P.T.C._2$, or from the $P.C._2$, proportional to the cubes of the distances, that at the *P.C.C.* being one half the offset. Or by the method in use with the Holbrook spirals; set up at the $P.C._2$, and measure the angle between the tangent at that point and the line to the $P.T.C._2$; take out the deflection corresponding to the flat curve for the entire distance, and assume the remainder to be due to the transition curve; intermediate deflections to be made up of the flat curve deflection proportional to distance, and the transition curve deflection proportional to the square of the distance.

Monuments should be placed at each end of each transition curve, so that the trackman may know where to begin his curvature and elevation of outer rail, and where to reach the full values required for the circular curve.

20. General Case.—The transition curve adds materially to the flexibility of the alignment, as already indicated.

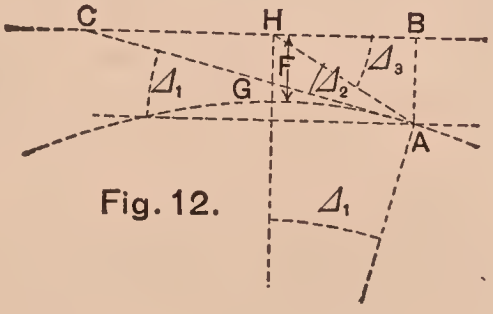

Fig. 12.

It also simplifies the field-work during location on crooked alignment by allowing, after the line has been fairly well fitted to the ground, any tangent or any curve to be studied by itself and shifted within the limits of the offsets if its position can be improved thereby. Many of the complicated field

problems can be simplified by running trial solutions; it often being easy to bring the last curve within the range of offsets of the tangent, and then use a transition curve. To find the *P.T.* and offset *F* between a circle and tangent which do not quite intersect or touch :

Run the curve to a convenient point *A*, Fig. 12, where it is approximately parallel to the given tangent; set up the instrument and measure the angle Δ_1 between the tangent proper and a parallel to the given tangent; also measure the perpendicular *AB*.

$$\text{Dist. to } P.T. = AG = \frac{\Delta_1}{D}.$$

$$\text{Offset } F = AB - AG \sin \frac{\Delta_1}{2}.$$

Or, sight from any point *A* near the *P.T.* to any point *H* on the tangent, and measure the angle Δ_2 between this line and the tangent proper at *A*; set up at *H* and measure the angle Δ_3 between this line and the given tangent; also measure the distance *AH*.

$$\Delta_1 = \Delta_3 \pm \Delta_2,$$

$$AB = AH \sin \Delta_3;$$

giving the same data as above.

EXAMPLE 1. A tangent joins a 6° curve to the left at the *P.T.* at sta. 824, and a 20° curve to the right at the *P.C.* at sta. 831. If the tangent be moved 3 ft. to the right at sta. 824, and 48.5 ft. to the left at sta. 831, to better fit the ground, find the data required.

Each central angle will be increased by $\tan^{-1} \frac{3 + 48.5}{700} = 4° 12\frac{1}{2}'$, lengthening the 6° curve by 70.1 ft., and the 20° curve by 21 ft.

The first offset $F = (R + 3) \cos 4° 12\frac{1}{2}' - R = 0.42.$

The second offset $F' = (R' + 48.5) \cos 4° 12\frac{1}{2}' - R' = 47.60.$

The length of the new tangent is best found when restaking it; it can be readily computed if desired.

21. Description of Right of Way.—The ordinary description for right of way calls for a given width each side of the center line, and then describes the center line as being so many feet along a tangent of a certain bearing, then along a curve of given degree, etc.

When a transition curve is added, it will be sufficient in describing the center line to note the offset thus: "Thence

along a tangent N. 20° W. from sta. 1081 to sta. 1086 + 50, a distance of 550 feet; thence along a 6° curve to the right to sta. 1091, a distance of 450 feet; thence along a tangent N. 7° E. to sta. 1102, a distance of 1100 feet, there being an offset of 0.4 feet at the beginning of the curve, and one of 0.5 feet at the end for a transition curve to connect the circular curve and tangent." The variation in transition curve for a given offset not being sufficient to seriously affect right-of-way lines.

FORMULAS.

(9), $I^\circ = \frac{l_1 D}{200} = 28.65 \frac{l_1}{R}$.

$$\phi^\circ = n^2 I^\circ = \frac{l^2}{{l_1}^2} I^\circ.$$

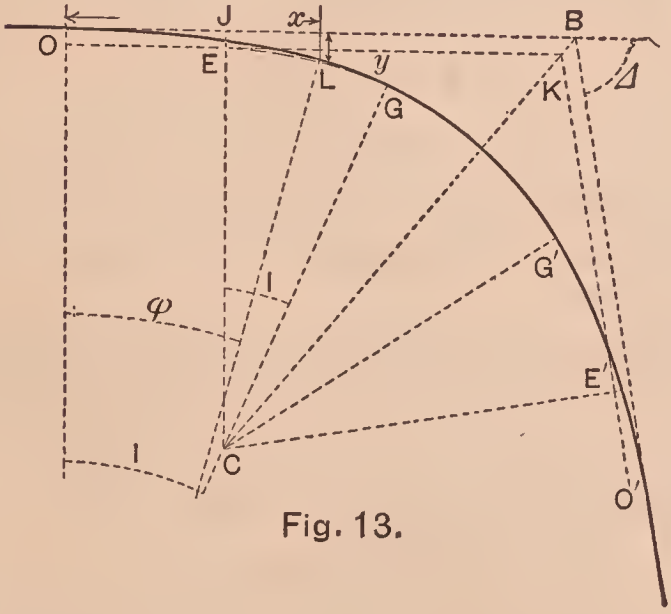

Fig. 13.

(4), $y = lC$; C from Table II, ϕ° as argument.
(6), $x = l - lE$; E from Table II, ϕ° as argument.
(5)–(10), x_1, y_1, F, x', y', e, e', $l' = x' + e$, see Table I.
(14)–(17), $d = \frac{I}{3} A - B$; A and B from Table III, n'', n, I° as arguments.

$$P.C \text{ to } P.C._1 \text{ (circular arc)} = \frac{I}{D} 100 = \frac{l_1}{2}.$$

COMPOUND CURVES.

(18), $l'' = l_1 \frac{D''}{D}$.

$$l_2 = l_1 - l'' = l_1 \frac{D - D''}{D}.$$

(19), $I'' = \frac{l'' D''}{200} = I \left(\frac{l''}{l_1}\right)^2$

(20), $\tan i = \frac{x' - x''}{F'' + R'' - F - R}$.

$P.T.C._2$ to $P.C.C.$ (circular arc) $= \frac{I - i}{D} 100.$

$P.C.C.$ to $P.C._2$ (circular arc) $= \frac{i - I''}{D''} 100.$

TABLES.

Table I.

l_1 = length of transition curve, § 3;
$I°$ = central angle of transition curve from $P.T.C.$ to $P.C._1$, (9);
= central angle of circular curve from $P.C.$ to $P.C._1$;

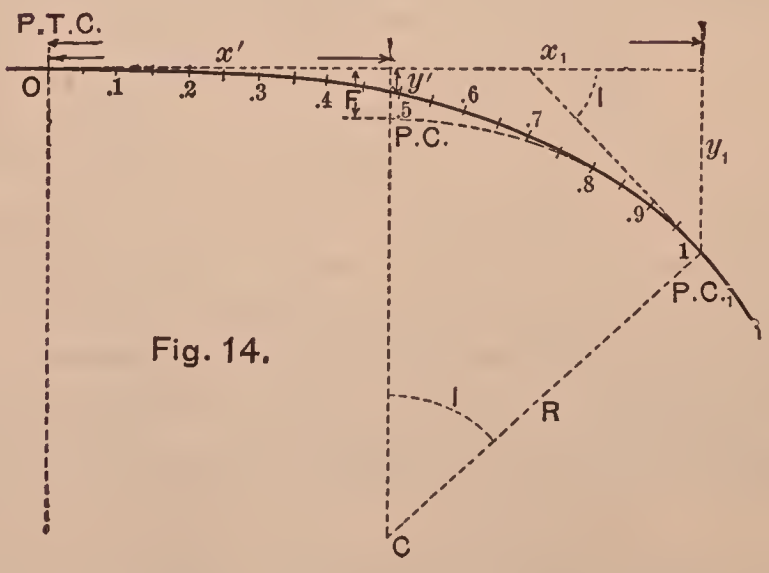

Fig. 14.

$x_1, y_1,$ = coördinates of $P.C._1$ (3)–(6);
$x', y',$ = coördinates of point opposite $P.C.$, (11);
F = offset from tangent to circular curve at $P.C.$, (11);
$l' = x' + e$ = length of transition curve from $P.T.C.$ to $P.C.$;
$e = l' - x'$ = excess of transition curve over tangent from $P.T.C.$ to $P.C.$;
$e' = \frac{l_1}{2} - l' -$ arc excess = excess of transition curve over circular curve from $P.C.$ to $P.C._1$, § 6.

It should be noted that for a given I linear dimensions are inversely as D. This will often allow one table to be used for interpolating in, or for extending the limits of, another. Thus for $l_1 = 50$, with $D = 4°$, halve the dimensions for $l_1 = 100$, with $D = 2°$, to obtain the data not given in the 4° table.

Table II.

$\phi° = n^2 I° =$ central angle of transition curve from $P.T.C.$ to point distant l from $P.T.C.$, where $n = \frac{l}{l_1}$;
$y =$ ordinate at any point from tangent at $P.T.C.$;
$x =$ abscissa at any point measured along tangent at $P.T.C.$

Table III.

n, ϕ, as above for point sighted.
$n'' =$ instrument point = distance from $P.T.C. \div l_1$.
$d =$ deflection angle from tangent at $P.T.C.$, in degrees, for all values of $I°$ from 0° to 60°, and for five values of n''. $I°$ is taken from Table I.

Table IV.

This is a table of actual deflections computed from Table III. The offsets F from the circular curve to the tangent at the $P.C.$ or $P.T.$ are given for easy reference, and the long chords, or distances from each chord point to the $P.T.C.$ at the flat or zero end of the transition curve, are given to aid in passing obstructions and to allow of passing from one end of the transition curve to the other with distance and alignment without stopping to run the curve.

It may be noted that the chord-point numbers are multiplied by the chord length to give the distances from the $P.T.C.$, and not by the total length of transition curve, as in Table III.

The deflection for any chord, or the angle which it makes with the tangent at the *P.T.C.*, is given at the intersection of the horizontal line bearing the chord-point number of one end with the vertical column bearing the chord-point number of the other end of the chord. To find the deflections for all the chords radiating from a given point, run out the horizontal line and down the vertical column bearing the point number.

Thus with the 50-foot-chord curve the deflections for the chords radiating from *C.P.*6 are: 6–0, 3° 00′; 6–1, 3° 35′; . . .; 6–12, 20° 59′.6; the tangent at 6, 9° 00′, coming from the *I* column.

Some of the chord-point columns have been omitted, which requires the use of double chord lengths if the instrument is not at the *P.T.C.* The chord lengths are limited nearly to those allowed for circular curves for the same curvature.

The range of the table can be extended, the same as for Table I, by remembering that for the same angles doubling distances will halve degrees of curvature, and *vice versa;* while the chord lengths will have the factor 4, because the doubling of distances divides the number of chords by 2, and *vice versa.* This can be seen by comparing the 100-foot chord curve with the 25, the 5-foot 4-degree with the 5-foot 16-degree, etc.; or in Table I the 2-degree with the 4-degree, etc.

Table V.

Radii found by dividing 5730 by *D*.

Deflection angles in minutes per foot of length of curve.

Table VI.

Actual circular arcs for a 100-foot chord, or for 100 feet measured along the shorter chords.

Table VII.

Tangents and externals for a 1° curve with radius = 5730.

All tables computed on the basis that $R = \frac{5730}{D}$, requiring 100-foot chords to be used up to 8° curve, 50-foot chords from 8° to 16° curves, 25-foot chords from 16° to 32°, 10-foot chords from 32° to 80°, and 5-foot chords for sharper curves.

In running centers for high-speed track where great precision is desired the chord lengths must be changed slightly for the curve to accurately fit its tangents as below.

CHORD CORRECTION, OR SUM OF CHORD CORRECTIONS per 100-ft. Station, for $R = 5730/D$. Actual arc = 100 .007 for the Circle and 100 for the Transition Curve.

Correction.	100-ft. Chords.	50-ft. Chords.	25-ft. Chords.	10-ft. Chords.	Tran. Cur. Corr.
+.007	0° 30′				.000
.006	1				−.001
.004	1 30				.003
+.002	2				.005
−.001	2 30				.008
.004	3				.011
.008	3 30	7°	14°	35°	.015
.013	4	8	16	40	.020
.018	4 30	9	18	45	.025
.024	5	10	20	50	.031
.031	5 30	11	22	55	.038
.038	6	12	24	60	.045
.046	6 30	13	26	65	.053
.055	7	14	28	70	.062
.064	7 30	15	30	75	.071
.074	8	16	32	80	.081

The table shows that a correction of .01 ft. for a 4° 30′ curve, with an increase of .01 for each additional ½° up to 8°, will be correct to about the nearest .01 for 100-ft. chords, with degree of curve inversely as chord length for shorter chords.

Curves sharper than 80° should usually be run on the radius basis like street railroad curves.

Table VIII.

Circular functions to five decimal places.

FORM FOR PRESERVING DEFLECTIONS MOST COMMONLY USED.

Deflections for 2° curve; $I = 1° 30'$; $l_1 = 150$ ft.; $F = .32$ ft.

n	⊙ at Sta. 0.	⊙ at Sta. 1.	⊙ at Sta. .5.
0	0° 0′	0° 30′	
.1			
.2			
.3			
.4			
.5	0 7½	0 52½	
.6			
.7			
.8			
.9			
1	0 30	1 30	

Deflections for 7° curve; $I = 10° .50$; $l_1 = 300$ ft.; $F = 4.57$ ft.

n	⊙ at Sta. 0.	⊙ at Sta. 1.	⊙ at Sta. .5.
0	0° 0′		0° 52½′
.1			
.2			
.25	0 13		1 32
.3			
.4			
.5	0 52½	6° 7½′	2 37½
.6			
.7			
.75	1 58		4 09½
.8			
.9			
1	3 30	10 30	6 07½

Deflections for 20° curve; $I = 60°$; $l_1 = 600$ ft.; $F = 50.33$ ft.

n	⊙ at Sta. 0.	⊙ at Sta. .75.	⊙ at Sta. 1
0	0° 0′	11° 13′	
.05	0 3	12 01	
.1	0 12	12 55½	
.15	0 27	13 56	
.2	0 48	15 02	
.25	1 15	16 14	
.3	1 48	17 32½	
.35	2 27	18 57	
.4	3 12	20 27	
.45	4 03	22 03	
.5	5 00	23 45	
.55	6 03	25 33	
.6	7 11½	27 27	
.65	8 26	29 27	
.7	9 47	31 33	
.75	11 13	33 45	46° 15′
.8	12 45	36 03	
.85	14 23	38 27	
.9	16 06	40 57	
.95	17 55	43 33	
1	19 48½	46 15	60 00

SUPPLEMENT.

22. Crossing Frogs.—These may be located on transition curves, especially for street-railroad work. In Fig. 15, let the tangent at the *P.T.C.* of the outer rail of a transition curve intersect the inner rail of a circular track at *A* at an angle *EAD*, and at a distance *AO* from the *P.T.C.*

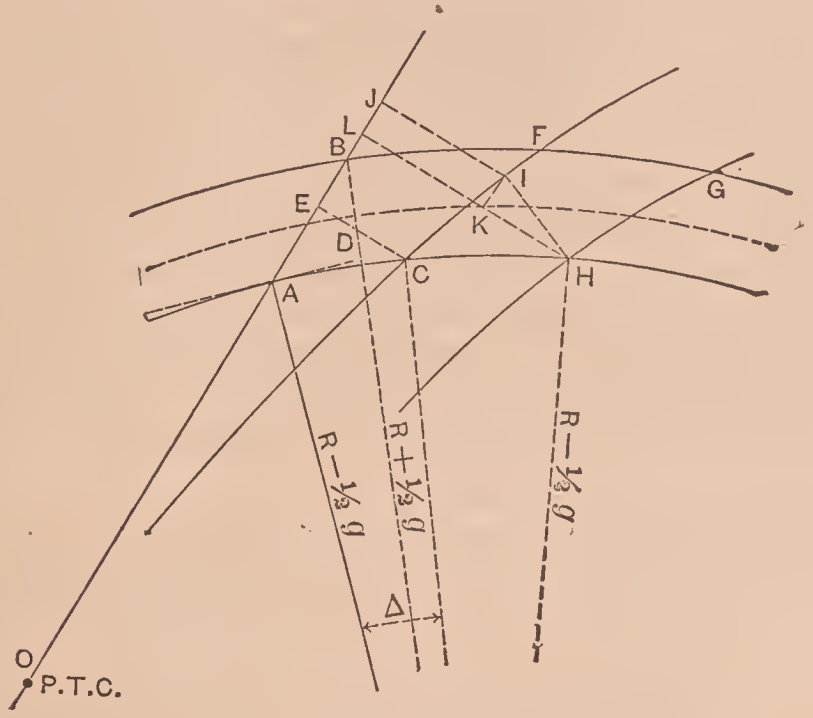

Fig. 15.

To find the frog C.—Plat to a scale of 5 feet to an inch and measure *AC*. Compute Δ from *AC* and the degree of the curve.

In the triangle *ACE*, $EAC = EAD + \frac{\Delta}{2}$, and the chord *AC* is known by scale. Find *AE* and *CE* by computation, which accurately locates the point *C* of the circle with reference to the *P.T.C.* of the transition. With the distance *OE* as x for the transition find ϕ and y by Table II*a*. This accurately locates the intersection of *EC* with the transition; its distance, $y - EC$, from *C* is the distance from the transition to the circle. Lay this distance off full size on *EC* and draw parallels to the sides of the frog at *C* to intersect, forming a corrective triangle. The side parallel to *AC* will give the

distance to add or subtract for a new AC. The second solution should give the desired intersection.

The frog angle at $C = BAD - \phi + \Delta$.

Distance $OC = x + e = l$, found by Table II.

Frog F.—Draw the radii at A and B, giving a triangle with the angle at A and sides $R - \frac{1}{2}g$ (g = gage) and $R + \frac{1}{2}g$ known. Having B, repeat the process for finding C.

Frog H.—Draw the normal HI to the transition; drop IJ perpendicular to AJ and scale AH and KHI or ϕ (ϕ can be better found by scaling OJ and computing by Table II*a*). Find the angle subtending the arc AH and the distances OL and HL, as for C. Find IK and HK from g and ϕ. Add IK to OL for x and enter Table II*a* for ϕ and y (if ϕ differs appreciably from the measured value, a corrected value of IK must be used). $y + HK$ compared with HL will give the distance between the two curves as before. Drawing the full-sized triangle of error, as at C, will give the corrected AH for a second approximation.

The difference between the l for F and that for C will give the distance CF between frogs.

If both tracks are on transition curves as in Fig. 16, find the x for the point C for one of the curves from the plot as

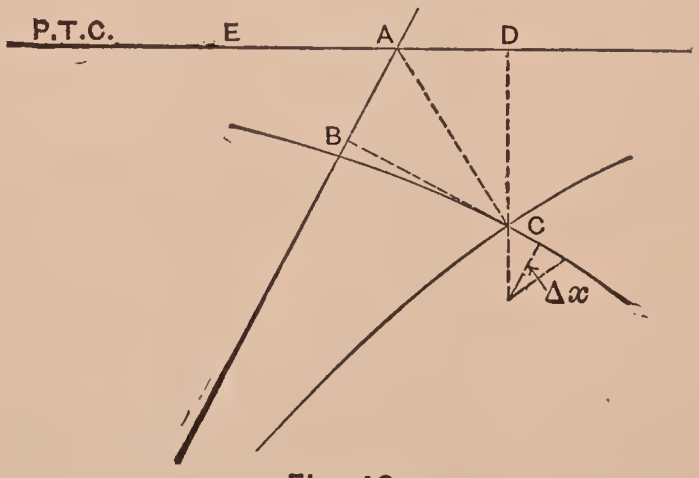

Fig. 16.

before and compute ϕ and y from Table II*a*. AB and BC would thus be known: compute AC and BAC. These with EAB allow AD and DC to be found. Adding AD to the distance of A from the $P.T.C.$ of the second transition will

give the x for the ordinate DC. Comparing DC with the y from Table IIa will give the distance between the two curves measured along DC. Laying this off full size on DC and drawing parallels to the curves at C will give the triangle of error as before, from which the correction to x (Δx) of the first curve can be found for a second approximation.

For those who prefer to work without the tables fairly simple relations between the coördinates x and y and the radius or degree of curve can be found by taking the first term of (3) and of (5), and eliminating k by (10). The central angle ϕ can be found from either y or x by using only the first term of the equation under (3) or under (5). With a slide-rule results are readily obtained and the approximate equations are accurate enough for many purposes.

For complicated street-railroad layouts a connecting curve made up of short circular arcs compounded is more common as it lends itself rather more readily to computation, but no difficulty need be experienced with the true transition curve along the lines as indicated above.

Example 1. Given Fig. 15, $AO = 112$ ft.; angle $EAD = 47° 01'$; 4° circular curve on center line; 200-ft. transition to a 20° curve on outer rail; to find the crossing frogs.

Frog C.—$AC = 6.2$ ft. from a plot to a scale of 5 ft. to an inch.

Δ for $AC = 4° \times .062 = 0°.248 = 0° 15'$. $\therefore BAC = 47° 8\frac{1}{2}'$.

$CE = 6.2 \sin 47° 8\frac{1}{2}' = 4.545$; $AE = 6.2 \cos 47° 8\frac{1}{2}' = 4.217$.

$x = 112 + 4.217 = 116.217$. I by (9) $= 20°$.

Table IIa, with $x = 116.217$, $I = 20$, gives $\phi = 6°.771$, $y = 4.580$.

$y - CE = 4.580 - 4.545 = .035$.

By laying .035 off on EC and completing the corrective triangle it is found that AC should be increased .055.

$AC = 6.255$ gives $\Delta = 0° 15'$; $BAC = 47° 8\frac{1}{2}'$.

$CE = 4.585$; $AE = 4.255$; $x = 116.255$.

This value of x gives $\phi = 6°.776 = 6° 46\frac{1}{2}'$; $y = 4.585$, $= CE$.

$\therefore$ Frog angle $= 47° 01' - (6° 46\frac{1}{2}') + (0° 15') = 40° 29\frac{1}{2}'$.

Frog distance from *P. T. C.* $= x + e = 116.255 + .163 = 116.42$ ft.

Frog H.—AH by scale $= 13.6$.

x by scale $= 121.8$, giving $\phi = 7°.45 = 7° 27'$.

Δ for $AH = 4 \times .136 = .544 = 0° 32\frac{1}{2}'$. $BAH = 47° 17'$.

$HL = 13.6 \sin 47° 17' = 9.992$; $AL = 13.6 \cos 47° 17' = 9.226$.

$IK = 4.71 \sin 7° 27' = .611$; $HK = 4.71 \cos 7° 27' = 4.670$.

$x = 112 + 9.226 + .611 = 121.837$, gives $\phi = 7°.447$; $y = 5.280$.

$y + HK = 9.950$. $9.950 - HL = -.042$.

.042 laid off on HL will decrease AH by scale .064.

$AH = 13.536$ gives $\Delta = 0° 32\frac{1}{2}'$; $BAH = 47° 17'$.

$HL = 9.945$; $AL = 9.182$; IK and HK as above.
$x = 121.793$, giving $\phi = 7°.442 = 7°\ 26\frac{1}{2}'$; $y = 5.275$.
$y + HK = 9.945, = IL$ and checking the frog-point.
$\therefore$ Frog angle $= 47°\ 01' - (7°\ 26\frac{1}{2}') + 0°\ 32\frac{1}{2}' = 40°\ 07'$.
Distance between frogs, $CH = 13.536 - 6.255 = 7.28$ ft.

Similarly the frogs at F and G can be found and the remaining distances. The distance between H and G can best be found by coördinates, having the distance of each from the tangent OJ and the projection of each upon this tangent.

Table Ia.

This is tabulated for radius, rather than for degree of circular curve, for convenience in running sharp street-railroad curves. The remarks for Table I, pp. 36, 37, apply, except that in comparing the length of the circular curve from $P.C.$ to $P.C._1$ with that of the corresponding portion of the transition curve the actual arcs ($I°R$ *arc* 1°, and $l_1 - l'$) are used, giving

$e' = \frac{l_1}{2} - l' =$ excess of transition over circular curve from $P.C.$ to $P.C._1$, § 6.

Table IIa.

This table is for coördinates when the abscissa x, rather than the distance l along the curve, is given.

$\phi =$ central angle of the transition curve between the $P.T.C.$ and the point with coördinates x and y, I being its largest value at the $P.C._1$.

$l_1 =$ length of transition curve for central angle I.

From (8), $\phi = I \frac{l^2}{l_1^2}$.

Putting in the value of l from (6),

$$\phi = \frac{I}{l_1^2} \frac{x^2}{(1-E)^2} = I \frac{x^2}{l_1^2} + I \frac{x^2}{l_1^2} F,$$

where $$1 + F = \frac{1}{(1-E)^2}.$$

From (4), $$y = lC, = \frac{C}{1-E} x = Gx.$$

F and G vary with ϕ as seen from the table.

TABLE I.

l_1	2° Curve.								3° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	.40	40	.09			.02	20	.01	.60	40	.13			.03	20	.02	40
60	.60	60	.21			5	30	3	.90	60	.31			8	30	4	60
80	.80	80	.37			9	40	5	1.20	80	.56			.14	40	7	80
100	1.00	100	.58			.14	50	7	1.50	100	.88			.22	50	.11	100
120	1.20	120	.84			.21	60	.10	1.80	120	1.26			.32	60	.16	120
140	1.40	140	1.14			.28	70	.14	2.10	140	1.71			.43	70	.21	140
160	1.60	160	1.49			.37	80	.19	2.40	160	2.23			.56	80	.28	160
180	1.80	180	1.89			.47	90	.24	2.70	180	2.82		.01	.71	90	.35	180
200	2.00	200	2.33			.58	100	.29	3.00	200	3.49		1	.87	100	.44	200
220	2.20	220	2.84			.70	110	.35	3.30	220	4.23		1	1.06	110	.53	220
240	2.40	240	3.35			.84	120	.42	3.60	239.9	5.03		1	1.26	120	.63	240
260	2.60	259.9	3.93		.01	.98	130	.49	3.90	259.9	5.90		2	1.48	130	.74	260
280	2.80	279.9	4 56		1	1.14	140	.57	4.20	279.8	6.84		2	1.71	140	.86	280
300	3.00	299.9	5.24		1	1.31	150	.65	4.50	299.8	7.85	.01	2	1.96	150	.98	300
320	3.20	319.9	5.96		2	1.49	160	.74	4.80	319.7	8.93	1	2	2.23	160	1.12	320
340	3.40	339.9	6.73		2	1.68	170	.84	5.10	339.7	10.08	1	3	2.52	170	1.26	340
360	3.60	359.9	7.54		2	1.88	180	.94	5.40	359.7	11.30	1	3	2.83	179.9	1.41	360
380	3.80	379.8	8.40	.01	2	2.10	190	1.05	5.70	379.6	12.59	1	4	3.15	189.9	1.57	380
400	4.00	399.8	9.30	1	3	2.32	200	1.16	6.00	399.6	13.95	1	5	3.49	199.9	1.74	400
420	4.20	419.8	10.25	1	4	2.56	210	1.28	6.30	419.5	15.38	2	7	3.85	209.9	1.92	420
440	4.40	439.7	11.26	1	4	2.81	219.9	1.41	6.60	439.4	16.88	2	8	4.22	219.9	2.11	440
460	4.60	459.7	12.31	1	4	3.08	229.9	1.54	6.90	459.4	18.45	2	9	4.61	229.9	2.31	460
480	4.80	479.7	13 40	1	5	3.35	239.9	1.68	7.20	479.3	20.08	2	.10	5.02	239.9	2.51	480
500	5.00	499.6	14.54	1	6	3.63	249.9	1.82	7.50	499.2	21.78	3	.11	5.45	249.9	2.72	500
520	5.20	519.6	15.72	1	7	3.93	259.9	1.96	7.80	519.0	23.56	3	.12	5.89	259.8	2.94	520
540	5.40	539.5	16.95	2	7	4.24	269.9	2.12	8.10	538.9	25.41	3	.14	6.35	269.8	3.18	540
560	5.60	559.5	18.23	2	8	4.56	279.9	2.28	8.40	558.8	27.32	4	.16	6.83	279.8	3.42	560
580	5.80	579.4	19.56	2	9	4.89	289.9	2.44	8.70	578.7	29.30	4	.18	7.33	289.8	3.66	580
600	6.00	599.3	20.93	2	9	5.23	299.9	2.61	9.00	598.5	31.36	5	.20	7.84	299.7	3.92	600
620	6.20	619.3	22.35	2	.10	5.59	309.9	2.79	9.30	618.4	33.48	6	.22	8.37	309.7	4.19	620
640	6.40	639.2	23.81	2	.11	5.95	319.8	2.97	9.60	638.2	35.67	6	.24	8.92	319.7	4.46	640
660	6.60	659.1	25.32	3	.12	6.33	329.8	3.16	9.90	658.0	37.93	6	.27	9.48	329.6	4.74	660
680	6.80	679.0	26.87	3	.13	6.72	339.8	3.36	10.20	677.8	40.26	7	.30	10.07	339.6	5.03	680
700	7.00	699.0	28.47	4	.14	7.12	349.8	3.56	10.50	697.6	42.66	7	.33	10.67	349.5	5.33	700

TABLE I.

l_1	4° Curve.								5° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	0.80	40	.18			.04	20	.02	1.00	40	.23		−.01	.06	20	.03	40
60	1.20	60	.42			.10	30	5	1.50	60	.52		1	.13	30	6	60
80	1.60	80	.74			.18	40	9	2.00	80	.93		1	.23	40	.12	80
100	2.00	100	1.16			.29	50	.14	2.50	100	1.45		1	.36	50	.18	100
120	2.40	120	1.67			.42	60	.21	3.00	120	2.09		1	.52	60	.26	120
140	2.80	140	2.28			.57	70	.28	3.50	140	2.85		1	.71	70	.36	140
160	3.20	160	2.98			.75	80	.37	4.00	159.9	3.72		−.01	.93	80	.47	160
180	3.60	179.9	3.77		.01	.94	90	.47	4.50	179.9	4.71			1.18	90	.59	180
200	4.00	199.9	4.65		1	1.16	100	.58	5.00	199.8	5.81			1.45	100	.73	200
220	4 40	219.9	5.62		1	1.41	110	.70	5.50	219.8	7.03	.01		1.76	110	.88	220
240	4.80	239.8	6.69	.01	2	1.67	120	.84	6 00	239.7	8.37	1	+.01	2.09	119.9	1.05	240
260	5.20	259.8	7.85	1	2	1.96	130	98	6.50	259.7	9.82	1	1	2.46	129.9	1.23	260
280	5.60	279.7	9.11	1	3	2.28	139.9	1.14	7.00	279.6	11.39	1	2	2.85	159.9	1.42	280
300	6.00	299.7	10.46	1	3	2.62	149.9	1.31	7.50	299.5	13.07	2	3	3.27	149.9	1 63	300
320	6.40	319.6	11.90	1	4	2.98	159.9	1.49	8.00	319.4	14.87	2	5	3.72	159.9	1.86	320
340	6.80	339.5	13.43	1	5	3.36	169.9	1.68	8.50	339.3	16.78	2	6	4.20	169.9	2.10	340
360	7.20	359.4	15.06	2	6	3.77	179.9	1.88	9.00	359.1	18.81	3	8	4.70	179.8	2.35	360
380	7.60	379.3	16.78	2	7	4.20	189.9	2.10	9.50	379.0	20.96	3	.10	5.24	189.8	2.62	380
400	8.00	399.2	18.59	2	9	4.65	199.9	2.32	10.00	398.8	23.22	3	.12	5.81	199.8	2.90	400
420	8.40	419.1	20.49	3	.11	5.12	209.8	2.56	10.50	418.6	25.59	4	.2	6.40	209.7	3.20	420
440	8.80	439.0	22.49	3	.14	5.62	219.8	2.81	11.00	438.4	28.08	5	.2	7.03	219.7	3.51	440
460	9.20	458.9	24.58	3	.15	6.15	229.8	3.07	11.50	458.1	30.69	6	.2	7.68	229.7	3.83	460
480	9.60	478.7	26.75	4	.17	6.69	239.8	3.34	12.00	477.9	33.40	7	.3	8.36	239.6	4.17	480
500	10.00	498.5	29.02	5	.20	7.26	249.7	3.63	12.50	497.7	36.23	7	.3	9.07	249.6	4.53	500
520	10.40	518.3	31.38	6	.23	7.85	259.7	3.92	13.00	517.3	39.18	8	.4	9.81	259.5	4.90	520
540	10.80	538.1	33.84	6	.25	8.47	269.6	4.23	13.50	537.0	42.24	9	.4	10.58	269.5	5.28	540
560	11.20	557.9	36.39	7	.26	9.11	279.6	4.55	14.00	556.7	45.41	.1	.4	11.37	279.4	5.67	560
580	11.60	577.6	39.03	7	.29	9.77	289.6	4.88	14.50	576.3	48.70	.1	.5	12.20	289.3	6.08	580
600	12.00	597.4	41.75	8	.32	10.45	299.5	5.22	15.00	595.9	52.10	.1	.5	13.05	299.3	6.51	600
620	12.40	617.1	44.57	9	.36	11.15	309.5	5.57	15.50	615.4	55.61	.1	.6	13.93	309.2	6.95	620
640	12.80	636.8	47.48	.1	.4	11.88	319.4	5.93	16.00	635.0	59.24	.2	.6	14.85	319.1	7.39	640
660	13.20	656.5	50.48	.1	.5	12.64	329.4	6.30	16.50	654.5	62.98	.2	.7	15.79	329.0	7.86	660
680	13.60	676.2	53.58	.1	.5	13.42	339.3	6.69	17.00	674.0	66.83	.2	.7	16.76	339.0	8.34	680
700	14 00	695.8	56.77	.1	.6	14.22	349.2	7.08	17.50	693.5	70.79	.2	.8	17.75	348.9	8.84	700

TABLE I.

l_1	6° Curve.							7° Curve.							l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	1.20	40	.28		−.01	.07	20	.03	1.40	40	.32		−.01	.08	20	.04	40
60	1 80	60	.63		1	.16	30	.08	2.10	60	.73		2	.18	30	.09	60
80	2.40	80	1.11		1	.28	40	.14	2.80	80	1.30		2	.33	40	.16	80
100	3.00	100	1.74		1	.44	50	.22	3.50	100	2.04		2	.51	50	.25	100
120	3.60	120	2.51		1	.63	60	.31	4.20	119.9	2.93		2	.73	60	.36	120
140	4.20	139.9	3 42		1	.86	70	.43	4 90	139.9	3.99		2	1.00	70	.50	140
160	4.80	159.9	4 47		1	1.12	80	.56	5.60	159 8	5.21		2	1.30	80	.65	160
180	5 40	179.8	5 66		1	1.42	90	.71	6.30	179.8	6.59	.01	2	1.65	90	.82	180
200	6.00	199.8	6.98	.01	−.01	1.75	100	.87	7.00	199.7	8.14	1	−.01	2.04	99.9	1.02	200
220	6 60	219.7	8.44	1		2.11	109.9	1.05	7.70	219.6	9.85	1		2.46	109.9	1.23	220
240	7.20	239.6	10.04	1	+.01	2.51	119.9	1.25	8.40	239.5	11.72	2		2.93	119.9	1.46	240
260	7.80	259.5	11.78	1	2	2.95	129.9	1.47	9.10	259.4	13 75	2	+.01	3.44	129.9	1.72	260
280	8.40	279.4	13.66	2	3	3.42	139.9	1.71	9.80	279.2	15.93	3	1	3.99	139.9	1.99	280
300	9.00	299.3	15.68	2	4	3.92	149.9	1.96	10.50	299.0	18.28	3	4	4.57	149.8	2.28	300
320	9.60	319.1	17 83	3	6	4.46	159.8	2.23	11.20	318.8	20.79	4	8	5.20	159 8	2.60	320
340	10.20	338.9	20.12	3	8	5.03	169.8	2.52	11.90	338.5	23.46	5	.11	5.87	169.7	2.93	340
360	10.80	358.7	22.56	4	.11	5.64	179.8	2 82	12.60	358.3	26.29	5	.14	6.58	179.7	3.28	360
380	11.40	378.5	25.13	5	.14	6.28	189.7	3.14	13.30	378.0	29.28	6	.17	7.33	189.7	3.66	380
400	12.00	398.2	27.83	5	.18	6.96	199.7	3.48	14.00	397.6	32.43	7	.22	8.12	199.6	4.05	400
420	12.60	418.0	30.68	6	.2	7.68	209.7	3.83	14.70	417.2	35.74	9	.3	8 95	209.5	4.46	420
440	13.20	437.7	33.66	7	.2	8.43	219.6	4.20	15.40	436.8	39.21	.10	.4	9.82	219.4	4.89	440
460	13.80	457.3	36.78	8	.3	9.21	229.5	4.59	16.10	456.4	42.84	.11	.4	10.73	229.4	5.35	460
480	14.40	477.0	40.03	9	.3	10.03	239.5	5.00	16.80	475.9	46.62	.13	.4	11.68	239.3	5.82	480
500	15.00	496.7	43.42	.10	.4	10.88	249.4	5.42	17.50	495.4	50.56	.14	.5	12.67	249.2	6.31	500
520	15.60	516.2	46.94	.1	.5	11.76	259.3	5.86	18.20	514.8	54.66	.2	.5	13.70	259.1	6.82	520
540	16.20	535.7	50.60	.1	.5	12.68	269.2	6 32	18.90	534.2	58 91	.2	.6	14.78	269.0	7.35	540
560	16.80	555.2	54.39	.2	.5	13.63	279.2	6.79	19.60	553.5	63.32	.2	.7	15.89	278.9	7.90	560
580	17.40	574.7	58.32	.2	.6	14.62	289.1	7.28	20.30	572.8	67 88	.2	.8	17.04	288.8	8.47	580
600	18.00	594.1	62.38	.2	.8	15.64	299.0	7.79	21.00	592.0	72.60	.3	.9	18.23	298.6	9.06	600
620	18.60	613.5	66.58	.2	.9	16.70	308.9	8.31	21.70	611.2	77.47	.3	1.0	19.46	308.5	9.67	620
640	19.20	632.8	70.91	.2	1.0	17.79	318.7	8.85	22.40	630.3	82.49	.3	1.1	20.73	318.4	10.29	640
660	19 80	652.1	75.37	.2	1.0	18.91	328.6	9.40	23.10	649.4	87.67	.3	1.2	22.04	328.3	10.94	660
680	20.40	671.4	79.97	.3	1.1	20.07	338.5	9.97	23.80	668.4	93.00	.4	1.3	23.39	338.1	11.60	680
700	21.00	690.6	84.70	.3	1.1	21.27	348.4	10.56	24.50	687.3	98.48	.4	1.5	24.78	347.9	12.28	700

TABLE I.

l_1	8° Curve.								9° Curve.							l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	1.60	40	.37			.09	20	.05	1.80	40	.42			.11	20	.05	40
60	2 40	60	.84			.21	30	.11	2.70	60	.94			.24	30	.12	60
80	3.20	80	1 49			.37	40	.19	3.60	80	1.67			.42	40	.21	80
100	4.00	100	2.33			.58	50	.29	4.50	99.9	2.62			.66	50	.33	100
120	4.80	119.9	3.35			.84	60	.42	5.40	119.9	3.77		.01	.94	60	.47	120
140	5.60	139.9	4.56		.01	1.14	70	.57	6.30	139.8	5.13	.01	1	1.28	70	.64	140
160	6.40	159.8	5.95	.01	2	1.49	80	.74	7.20	159.7	6.70	1	2	1.68	80	.84	160
180	7.20	179.7	7.53	1	3	1.88	89.9	.94	8.10	179.6	8.47	1	3	2.12	89 9	1.06	180
200	8.00	199.6	9.29	1	4	2.32	99.9	1.16	9.00	199.5	10.45	2	4	2.61	99.9	1.30	200
220	8.80	219.5	11.24	2	6	2.81	109.9	1.40	9.90	219.3	12.64	2	6	3.16	109.9	1.58	220
240	9.60	239.3	13 38	2	7	3 35	119.9	1.67	10.80	239.2	15.04	3	9	3.76	119.9	1.88	240
260	10.40	259 1	15.70	3	.10	3.93	129.8	1.96	11.70	258.9	17.64	3	.12	4.42	129.8	2.20	260
280	11.20	278.9	18.20	3	.13	4.56	139.8	2.27	12.60	278.7	20.45	4	.16	5.12	139.8	2.55	280
300	12.00	298.7	20.88	4	.16	5.23	149.8	2.61	13.50	298.3	23.47	5	.20	5.88	149.7	2.93	300
320	12.80	318.4	23.74	5	.20	5.95	159.7	2.97	14.40	318.0	26.69	6	.24	6.69	159.7	3.33	320
340	13.60	338.1	26.79	6	.24	6.71	169.7	3.35	15.30	337.6	30.11	8	.30	7.55	169.6	3.76	340
360	14.40	357.7	30.02	7	.29	7.52	179.6	3.75	16.20	357.1	33.73	9	.36	8.45	179.5	4.21	360
380	15.20	377.3	33.43	8	.34	8.37	189.5	4.17	17.10	376.6	37.56	.11	.43	9.41	189.4	4 69	380
400	16.00	396.9	37.02	.10	.41	9.27	199.5	4.62	18.00	396.1	41.59	.12	.51	10.43	199.3	5.19	400
420	16.80	416.4	40.79	.1	.5	10.22	209.4	5.09	18.90	415.4	45 82	.1	.6	11.49	209.2	5.72	420
440	17.60	435.9	44 74	.1	.6	11.21	219.3	5.58	19.80	434.7	50.25	.2	.6	12.61	219.1	6.27	440
460	18 40	455.3	48.87	.1	.7	12.25	229.2	6.10	20.70	454.0	54.88	.2	.7	13.78	229.0	6.84	460
480	19.20	474.6	53.18	2	.7	13.34	239.1	6.64	21.60	473.2	59.70	.2	.9	14.99	238.8	7.41	480
500	20.00	493.9	57.66	.2	.8	14.47	249.0	7.19	22.50	492.3	64.72	.2	1.0	16.26	248.7	8.07	500
520	20.80	513.2	62 32	.2	.9	15.64	258.8	7.77	23.40	511.4	69.94	.3	1.1	17.58	258.5	8.72	520
540	21.60	532.4	67.16	.2	1.0	16.86	268.7	8.37	24 30	530.4	75.35	.3	1.3	18.95	268.3	9.39	540
560	22.40	551.5	72.18	.3	1.0	18.13	278.6	9.00	25.20	549.2	80.96	.3	1.5	20.37	278.1	10.08	560
580	23.20	570.6	77.37	.3	1.2	19.45	288.4	9.65	26.10	568.0	86.76	.4	1.6	21.84	287.9	10.80	580
600	24.00	589.6	82.73	.3	1.4	20.81	298.2	10.31	27.00	586.7	92.75	.4	1 8	23.36	297.7	11.55	600
620	24.80	608.5	88.26	.4	1.4	22.21	308.1	11.00	27.90	605.3	98.93	.5	2.0	24.93	307.4	12.31	620
640	25.60	627.3	93.96	.4	1.6	23.66	317.9	11.71	28.80	624.0	105.30	.5	2.2	26.55	317 2	13.10	640
660	26.40	646.1	99.84	.4	1.9	25.15	327.6	12.44	29.70	642.5	111.86	.6	2.3	28.22	327.0	13.92	660
680	27.20	664.8	105.88	.5	2.0	26.68	337.4	13.19	20.60	660.9	118.60	.6	2.5	29.94	336.8	14.76	680
700	28.00	683.5	112.09	.5	2.2	28.25	347.2	12.96	31.50	679.1	125.53	.7	2.7	31.71	346.5	15.62	700

TABLE I.

l_1	10° Curve.								11° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	2.00	40	.47			.12	20	.06	2 20	40	.51			.13	20	.06	40
60	3.00	60	1.05			.26	30	.13	3.30	60	1.15			.29	30	.14	60
80	4.00	80	1 86			.47	40	.23	4.40	80	2.05			.51	40	.26	80
100	5.00	99.9	2.91			.73	50	.36	5.50	99.9	3.20			.80	50	.40	100
120	6.00	119.9	4 19		.01	1.05	60	.52	6 60	119.8	4.60	.01		1.15	60	.57	120
140	7.00	139.8	5.70	.01	1	1.43	70	.71	7.70	139.7	6.25	1	.01	1.56	70	.78	140
160	8.00	159.7	7.44	1	2	1.86	79.9	93	8.80	159.6	8.16	1	3	2.04	79 9	1.02	160
180	9.00	179.6	9.41	1	4	2.36	89.9	1.18	9.90	179.5	10.33	1	5	2.58	89.9	1.29	180
200	10.00	199.4	11.61	2	6	2.91	99.9	1.45	11.00	199.3	12.75	2	7	3.19	99.9	1.59	200
220	11.00	219.2	14.04	3	8	3.51	109 9	1.75	12.10	219.0	15.43	3	.10	3.86	109.8	1.93	220
240	12.00	239.0	16.70	3	.11	4.18	119.8	2.09	13.20	238.7	18.36	4	.13	4.60	119.8	2.30	240
260	13.00	258.7	19.59	4	.15	4.90	129.8	2.45	14.30	258.4	21.54	5	.18	5.40	129.7	2.69	260
280	14.00	278.3	22.71	5	.19	5.68	139.7	2.84	15.40	278.0	24.96	6	.23	6.25	139 7	3 12	280
300	15.00	297.9	26.05	6	.24	6.52	149.7	3.26	16.50	297.5	28.63	8	.28	7.17	149.6	3.58	300
320	16 00	317.5	29.62	8	.29	7 42	159.6	3.70	17.60	317.0	32.55	.10	.35	8.15	159.5	4.07	320
340	17.00	337.0	33.41	9	.37	8 37	169.5	4.17	18.70	336.4	36.71	.11	.44	9.20	169.4	4.59	340
360	18.00	356.5	37.43	.11	.44	9.38	179.4	4.67	19.80	355 7	41.11	.13	.52	10.31	179.3	5.13	360
380	19 00	375.9	41.67	.13	.52	10.45	189.3	5.20	20.90	375.0	45.76	.15	.62	11 48	189.2	5 70	380
400	20.00	395.2	46.13	.15	.61	11.57	199.2	5.76	22.00	394.2	50.65	.18	.73	12.72	199.0	6.31	400
420	21.00	414.5	50.81	.2	.7	12.75	209.1	6.34	23.10	413.3	55.78	.2	.8	14.01	208.9	6.95	420
440	22.00	433.6	55.72	.2	.8	13.99	218.9	6.95	24.20	432.2	61.15	.2	1.0	15.37	218.7	7.62	440
460	23.00	452.6	60.84	.2	.9	15.29	228.7	7.59	25.30	451.1	66.76	.3	1.1	16.79	228.5	8.33	460
480	24.00	471.7	66.18	.3	1.1	16.64	238.6	8.25	26.40	469.9	72 61	.3	1.3	18.28	238.3	9.06	480
500	25.00	490.6	71.73	.3	1.2	18.05	248.4	8.94	27.50	488.6	78.69	.4	1.4	19.83	248.1	9.81	500
520	26.00	509.4	77 50	.3	1.3	19.51	258 2	9.66	28.60	507.2	85.00	.4	1.6	21.44	257.9	10.59	520
540	27.00	528.1	83.48	.4	1.5	21 03	268.0	10.40	29 70	525.7	91.53	.4	1.8	23.11	267.6	11.40	540
560	28 00	546.8	89.67	.4	1.7	22.60	277.8	11.17	30.80	544.1	98.29	.5	2.0	24.83	277.4	12.24	560
580	29.00	565.3	96.07	.5	2.0	24.23	287.5	11.97	31.90	562.4	105.27	.6	2.3	26.60	287.1	13.11	580
600	30.00	583.8	102.68	.5	2.2	25.91	297.3	12.79	33.00	580.5	112.48	.6	2.5	28.44	296.8	14.01	600
620	31.00	602.1	109.49	.6	2.4	27.64	307.0	13.64	34.10	598 5	119.91	.7	2.7	30.34	306.5	14.93	620
640	32.00	620.3	116.51	.6	2.6	29.44	316.7	14.51	35.20	616.4	127.56	.7	3.0	32.31	316 1	15.87	640
660	33.00	638.4	123.73	.7	2.9	31.29	326.4	15.40	36.30	634.1	135.43	.8	3.3	34.34	325.7	16.84	660
680	34.00	656.5	131.15	.7	3.2	33.19	336.0	16.31	37.40	651.7	143.51	.9	3.7	36.42	335.3	17.84	680
700	35.00	674.3	1 8.77	.8	3.4	35.14	345.7	17.25	38 50	669 2	151.79	.9	4.0	38.56	341.9	18.87	700

TABLE I.

l_1	12° Curve.							13° Curve.							l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	2.40	40	.56			.14	20	.07	2 60	40	.60			.15	20	.07	40
60	3.60	60	1.26			.32	30	.16	3.90	60	1.36			.34	30	.17	60
80	4.80	79.9	2.23			.56	40	.28	5.20	79.9	2.42			.61	40	.30	80
100	6.00	99.9	3.49			.87	50	.44	6.50	99.9	3.78			.95	50	.47	100
120	7.20	119.8	5.02	.01		1.26	60	.63	7.80	119.8	5.44	.01		1.36	60	.68	120
140	8.40	139.7	6.83	1	.01	1.71	69.9	.85	9.10	139.6	7.40	1	.02	1.85	69.9	.92	140
160	9.60	159.6	8.92	1	3	2.23	79.9	1.11	10.40	159.5	9.66	2	4	2.42	79.9	1.21	160
180	10.80	179.4	11.28	2	5	2.82	89 9	1.41	11.70	179.3	12.22	2	6	3.06	89.8	1.53	180
200	12.00	199.1	13.92	3	8	3.49	99.8	1.74	13.00	199.0	15.07	3	.10	3.77	99.8	1.88	200
220	13.20	218.8	16.83	4	.11	4.22	109.8	2.10	14.30	218.6	18.22	4	.13	4.56	109.8	2.28	220
240	14.40	238.5	20.01	5	.15	5.01	119.7	2.50	15.60	238.2	21.66	6	.18	5.42	119.7	2.71	240
260	15.60	258.1	23.47	6	.20	5.88	129.7	2.93	16.90	257.7	25.39	7	.24	6.36	129.6	3.17	260
280	16.80	277.6	27.20	8	.27	6.82	139.6	3.40	18.20	277.2	29.43	9	.31	7.38	139.5	3.68	280
300	18.00	297.0	31.20	9	.34	7.82	149.5	3.90	19.50	296 5	33.76	.11	.39	8.48	149.4	4.22	300
320	19.20	316.4	35.46	.11	.42	8.89	159.4	4.43	20.80	315.8	38.37	.13	.49	9.64	159.3	4.79	320
340	20.40	335.7	39.98	.13	.52	10.03	169.3	4.99	22 10	335.0	43.26	.16	.60	10.87	169.2	5.40	340
360	21.60	354.9	44.77	.15	.62	11.24	179.1	5.59	23.40	354.1	48.43	.19	.72	12.18	179 0	6.04	360
380	22.80	374.0	49 83	.18	.75	12.52	189.0	6.22	24.70	373 0	53.88	.22	.86	13.56	188 8	6.72	380
400	24.00	393.0	55.15	.21	.88	13.86	198.8	6.87	26.00	391.8	59.61	.25	1.03	15.01	198.6	7.43	400
420	25.20	411.9	60.73	.2	1.0	15.27	208.6	7.57	27.30	410.5	65 62	.3	1.2	16 53	208.4	8.18	420
440	26.40	430.8	66 56	.3	1 2	16.75	218.4	8.29	28.60	429.2	71.91	.3	1.4	18.13	218.2	8.96	440
460	27.60	449.4	72 64	.3	1.4	18.30	228.2	9.05	29.90	447.6	78.47	.4	1.6	19 80	227.9	9.77	460
480	28.80	468.0	78.98	.4	1.6	19.92	238.0	9.84	31.20	466.0	85.29	.4	1 8	21.54	237.6	10.62	480
500	30.00	486.4	85.57	.4	1.8	21.60	247.7	10.65	32.50	484 2	92.38	.5	2.1	23.35	247.3	11.50	500
520	31.20	504.8	92.40	.5	2.0	23.34	257.4	11.49	33.80	502.2	99 73	.5	2.3	25.23	257.0	12.41	520
540	32.40	523.0	99.48	.5	2 2	25.15	267.1	12.37	35.10	520.1	107.34	.6	2.6	27.18	266.7	13.35	540
560	33.60	541.0	106.80	.6	2 5	27.02	276.8	13.28	36.40	537.8	115.21	.7	2.9	29.21	276.3	14.32	560
580	34.80	559.0	114.36	.6	2.8	28.96	286.4	14.21	37.70	555.4	123.33	.7	3.2	31.31	285.9	15.32	580
600	36.00	576.8	122.16	.7	3.1	30.96	296.1	15.17	39.00	572.8	131.70	.8	3.6	33.47	295.4	16.35	600
620	37.20	594.4	130.19	.8	3 4	33.03	305.7	16.16	40.30	590.0	140.31	.9	4.0	35.70	304.9	17.41	620
640	38.40	611.9	138.45	.8	3.8	35.16	315.2	17.18	41.60	607.1	149.16	1 0	4.4	38.00	314 4	18.50	640
660	39.60	629.2	146.93	.9	4.1	37.35	324.8	18.23	42.90	624 0	158.23	1.1	4.8	40.36	323 9	19.62	660
680	40.80	646.3	155 64	1.0	4.5	39 60	334.3	19.30	44.20	640.6	167.56	1.2	5.3	42.79	333.3	20.77	680
700	42.00	663.3	164.58	1.1	5.0	41.92	343.8	20.40	45.50	657.1	177.11	1.3	5.8	45.28	342.8	21.95	700

TABLE I.

l_1	14° Curve.								16° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	2.8	40	.65			.16	20	.08	3.2	40	.74			.19	20	.09	40
60	4.2	60	1.47			.37	30	.18	4.8	60	1.67			.42	30	.21	60
80	5.6	79.9	2.60			.65	40	.32	6.4	79.9	2.98			.75	40	.37	80
100	7.0	99 8	4.07			1.02	50	.51	8.0	99.8	4.65	.01	.02	1.16	50	.58	100
120	8.4	119.7	5.86	.01		1.47	60	.73	9.6	119.7	6.69	1	4	1.67	59.9	.83	120
140	9.8	139.6	7.97	1	.02	1.99	69 9	.99	11.2	139.5	9.10	2	7	2.27	69.9	1.13	140
160	11.2	159.4	10.40	2	4	2.60	79.9	1.30	12.8	159 2	11.87	2	.10	2.97	79.9	1.48	160
180	12.6	179.1	13.15	3	6	3.29	89.8	1.64	14.4	178.9	15.01	4	.14	3.76	89.8	1.87	180
200	14.0	198.8	16.22	4	.10	4.06	99.8	2.03	16.0	198 5	18.51	5	.20	4.64	99.7	2.31	200
220	15.4	218.4	19.61	5	.15	4.92	109.7	2.45	17.6	217.9	22.37	6	.27	5.61	109.7	2.79	220
240	16.8	237.9	23.31	6	.22	5.85	119.7	2.91	19.2	237.3	26.59	8	.35	6.68	119.5	3.32	240
260	18.2	257.4	27.33	8	.29	6.86	129.5	3.41	20.8	256.6	31.16	.10	.42	7.83	129.4	3.89	260
280	19.6	276.7	31.66	.11	.36	7.95	139.4	3.95	22.4	275.8	36.09	.13	.50	9.07	139.3	4.50	280
300	21.0	296.0	36.30	.13	.46	9.12	149.3	4.53	24.0	294.8	41.37	.16	.63	10.40	149.1	5.15	300
320	22.4	315.1	41.24	.2	.6	10.36	159.2	5.14	25.6	313.7	46.99	.2	.8	11.82	158.9	5.85	320
340	23.8	334.2	46.49	.2	.7	11.68	169.0	5.79	27.2	332.4	52.95	.2	1.0	13.33	168.7	6.59	340
360	25.2	353.1	52 04	.2	.8	13.09	178.8	6.48	28.8	351.0	59.24	.3	1.2	14.93	178.5	7.38	360
380	26.6	371.9	57.89	.2	1.0	14.58	188.6	7.21	30.4	369.4	65.86	.3	1.4	16.62	188.2	8.20	380
400	28.0	390.6	64.05	.3	1.2	16.14	198.4	7.98	32.0	387.7	72.82	.4	1.7	18 40	197.9	9.06	400
420	29.4	409.1	70 50	3	1.4	17.78	208.2	8.78	33.6	405.8	80.10	.4	1.8	20.26	207.6	9.96	420
440	30.8	427.5	77.23	.4	1.6	19.50	217.9	9.62	35.2	423.7	87.70	.5	2.2	22.21	217.3	10.90	440
460	32.2	445.7	84.24	.4	1.8	21.29	227.6	10.49	36.8	441 4	95.61	.6	2.5	24.25	226 9	11.88	460
480	33.6	463.8	91.54	.5	2.1	23.16	237.3	11.39	38.4	458.9	103.84	.6	2.9	26.37	236.4	12.90	480
500	35.0	481.7	99.12	.6	2.4	25.11	246.9	12.32	40.0	476.2	112.37	.7	3.3	28.58	246.0	13.95	500
520	36.4	499.4	106.98	.6	2.7	27.13	256.5	13.29	41.6	493.3	121.19	.8	3.7	30.87	255.5	15.04	520
540	37 8	517.0	115.11	.7	3.0	29.22	266.1	14.29	43.2	510.1	130.30	.9	4.1	33.24	265.0	16.16	540
560	39 2	534.4	123.50	.8	3.4	31.39	275.7	15.33	44.8	526.7	139.70	1.0	4.6	35.69	274.4	17.32	560
580	40.6	551.6	132.15	.8	3.8	33.63	285.2	16.40	46.4	543.1	149.38	1.1	5.1	38.23	283.8	18.51	580
600	42.0	568.6	141.07	.9	4.2	35.94	294.7	17.50	48.0	559.2	159.33	1.2	5.6	40.84	293.1	19.72	600
620	43.4	585.4	150.24	1.0	4.6	38.33	304.2	18.63	49.6	575.1	169.55	1.3	6.2	43.53	302.4	20.96	620
640	44.8	602.0	159.66	1.1	5.1	40.79	313.6	19.79	51.2	590.8	180.03	1.4	6.8	46.30	311.7	22.24	640
660	46.2	618 4	169.32	1.3	5 5	43.32	323.0	20.97	52.8	606.1	190.76	1.6	7.4	49.15	320.9	23.55	660
680	47.6	634.6	179 22	1.4	6.1	45.92	332.3	22.18	54.4	621.2	201.73	1.7	8.2	52.08	330.0	24.89	680
700	49.0	650 5	189.35	1 5	6.7	48.58	341.6	23.42	56.0	636.0	212.95	1.8	9.0	55.09	339.1	26 25	700

TABLE I.

l_1	18° Curve.								20° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	3.6	40	.84			.21	20	.10	4.0	40	.93			.23	20	.12	40
60	5.4	59.9	1 88			.47	30	.24	6.0	59.9	2.09			.52	30	.26	60
80	7.2	79.9	3.35			.84	40	.42	8.0	79.8	3.72		.01	.93	40	0.46	80
100	9.0	99.8	5.23	.01	.02	1.31	50	.65	10.0	99.7	5.81	.01	2	1.45	49.9	0.72	100
120	10.8	119.6	7.52	1	5	1.88	59.9	.94	12.0	119.5	8.35	2	6	2.09	59.9	1.04	120
140	12.6	139.3	10.22	2	8	2.56	69.9	1.28	14.0	139.2	11.35	3	.10	2.84	69.8	1.42	140
160	14.4	159.0	13 34	3	.13	3.34	79.8	1.67	16.0	158.8	14 81	4	.15	3.71	79.8	1.85	160
180	16.2	178.6	16.87	4	.19	4.23	89.8	2.11	18.0	178.2	18.72	6	.22	4 70	89.7	2.34	180
200	18.0	198.0	20.80	6	.25	5.22	99.7	2.60	20.0	197.6	23.07	8	.30	5.79	99.6	2.88	200
220	19.8	217.4	25 13	8	.33	6.31	109.6	3.14	22.0	216.8	27.86	.10	.41	7 00	109.5	3.48	220
240	21.6	236.6	29.85	.10	.44	7.50	119.4	3.73	24.0	235.8	33.09	.13	.54	8.31	119.3	4.13	240
260	23.4	255.7	34.97	.13	.57	8.79	129.3	4.37	26.0	254.7	38.75	.16	.70	9.75	129.1	4.83	260
280	25.2	274.6	40.48	.17	.71	10.18	139.1	5.05	28.0	273.4	44.84	.20	.88	11.30	138.9	5.58	280
300	27.0	293.4	46.37	.20	.87	11.67	148.9	5.78	30.0	291.9	51.35	.25	1.07	12.96	148.6	6.39	300
320	28.8	312.0	52.65	.2	1.1	13.27	158.7	6.56	32.0	310.2	58.27	.3	1.3	14.73	158.3	7.25	320
340	30.6	330.4	59.30	.3	1.3	14.97	168.4	7.38	34.0	328.2	65.58	.4	1.6	16 60	168.0	8 16	340
360	32 4	348.7	66.32	.3	1.5	16.76	178.1	8.25	36.0	346.0	73.30	.4	1.9	18.58	177.6	9.11	360
380	34.2	366.7	73.70	.4	1.8	18.65	187.8	9.16	38.0	363.6	81.40	.5	2.2	20.67	187.2	10.11	380
400	36.0	384.5	81.43	.5	2.1	20.63	197.4	10.12	40.0	380.9	89.89	.6	2.6	22.86	196.8	11.15	400
420	37.8	402.1	89 51	.5	2.4	22.71	207.0	11.12	42.0	398.0	98.75	.7	3.0	25.16	206 3	12.24	420
440	39.6	419.5	97.94	.6	2.8	24.89	216.5	12.16	44.0	414.8	107.97	.7	3.5	27.56	215.7	13.38	440
460	41.4	436.6	106.72	.7	3.2	27.17	226.1	13.24	46.0	431.2	117.54	.8	3.9	30.06	225.1	14.56	460
480	43 2	453.4	115.83	.8	3.6	29.54	235.5	14.36	48.0	447.4	127 46	1.0	4.5	32 67	234.5	15.77	480
500	45.0	470.0	125.24	.9	4.1	32.00	245.0	15.52	50.0	463 2	137.71	1.1	5.1	35.37	243.8	17.02	500
520	46.8	486 4	134 97	1.0	4.6	34.55	254.3	16.71	52.0	478.8	148.29	1.2	5.7	38.17	253.0	18.31	520
540	48.6	502 4	145.01	1.1	5.2	37.19	263.7	17.94	54.0	494.0	159.17	1.3	6.4	41.07	262.2	19.63	540
560	50.4	518.2	155.34	1.2	5.8	39 92	272.9	19.20	56.0	508.8	170.36	1.5	7.2	44.07	271.3	20.99	560
580	52.2	533.7	165.96	1.3	6.4	42.74	282.2	20.50	58.0	523.3	181 83	1.6	8 0	47.16	280.4	22.39	580
600	54.0	548.9	176.86	1.5	7.1	45.64	291.4	21.83	60.0	537.5	193.58	1.7	8.8	50.33	289.4	23.81	600
620	55 8	563.8	188.03	1.6	7.8	48.63	300.5	23.20	62.0	551.2	205 60	1 8	9.8	53.60	298.3	25.26	620
640	57.6	578.4	199.46	1.7	8.6	51 70	309.6	24.59	64.0	564.6	217.87	2.1	10.7	56 96	307.1	26.74	640
660	59.4	592.6	211.15	1.9	9.4	54.85	318.6	26.00	66.0	577 6	230.38	2.3	11.7	60.41	315.9	28.25	660
680	61.2	606 5	223.09	2.1	10.3	58 09	327.6	27.45	68 0	590.3	243.12	2.4	12.8	63.94	324.6	29.77	680
700	63.0	620.1	235.28	2.2	11.2	61.40	336.5	28.93	70.0	602.5	256.07	2.6	14.0	67.56	333.3	31.32	700

TABLE I.

l_1	22° Curve.								24° Curve.							l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	4.4	40	1.02			.26	20	.13	4.8	40	1.12			.28	20	.14	40
60	6.6	59.9	2.30			.58	30	.29	7.2	59.9	2.51			.63	30	.31	60
80	8.8	79.8	4.09		.01	1.02	40	51	9.6	79.8	4.46	.01	.02	1.12	40	56	80
100	11.0	99.6	6.38	.01	2	1.60	49.9	80	12.0	99.6	6.96	1	4	1.74	49.9	.87	100
120	13.2	119.4	9.18	2	7	2.30	59.9	1.15	14.4	119 2	10.01	2	8	2.51	59.9	1.25	120
140	15.4	139.0	12.48	3	.12	3.13	69.8	1.56	16.8	138.8	13.60	4	14	3.41	69.8	1.70	140
160	17.6	158.5	16.27	4	.18	4.08	79.7	2.03	19.2	158.2	17.73	5	.21	4.45	79.7	2.21	160
180	19.8	177.9	20.56	6	.27	5.16	89.6	2.57	21.6	177.5	22.39	8	.31	5.62	89.6	2.79	180
200	22.0	197.1	25.33	9	.37	6.37	99.5	3.16	24.0	196.5	27.57	.11	.42	6.93	99.4	3.44	200
220	24.2	216.1	30.58	.12	.50	7.69	109.3	3.81	26.4	215.4	33 27	.15	.58	8.38	109.2	4.15	220
240	26.4	235.0	36.30	.15	.65	9.13	119.1	4.53	28.8	234.0	39.49	.18	.77	9.97	119.0	4.92	240
260	28.6	253.6	42.49	.19	.83	10.71	128.9	5.30	31.2	252.4	46.20	.23	.99	11.67	128.7	5.75	260
280	30.8	272.0	49.14	.24	1.05	12.42	138.6	6.12	33.6	270.5	53.40	.29	1.25	13.51	138.4	6.64	280
300	33.0	290.2	56.24	.29	1.30	14.22	148.3	7.00	36.0	288.4	61.08	.35	1.54	15.48	148.0	7.59	300
320	35.2	308.1	63.78	.4	1.6	16.15	158.0	7.93	38.4	305.9	69.22	.4	1.9	17.58	157.6	8.59	320
340	37.4	325.8	71.75	.4	1.9	18.21	167.6	8.92	40.8	323.2	77.82	.5	2.3	19.81	167.2	9.65	340
360	39.6	343.2	80.15	.5	2.3	20.38	177.2	9.95	43.2	340.1	86 86	.6	2.7	22.15	176.6	10.77	360
380	41.8	360.3	88.95	.6	2.7	22.66	186.7	11.03	45.6	356.6	96.33	.7	3.2	24.63	186.0	11.93	380
400	44.0	377.0	98.15	.7	3.1	25.05	196.1	12.17	48.0	372.8	106.22	.8	3.7	27.22	195.4	13.14	400
420	46.2	393.5	107.75	.8	3.6	27.56	205.5	13.34	50.4	388.6	116.50	.9	4.3	29.94	204.7	14.40	420
440	48.4	409.6	117.72	.9	4.2	30.18	214.9	14.56	52.8	404.1	127.17	1.0	4.9	32.77	213.9	15.70	440
460	50.6	425.4	128.05	1.0	4.8	32.92	224.1	15.82	55.2	419.1	138.21	1.2	5.6	35.72	223.1	17.05	460
480	52.8	440.8	138.73	1.1	5.5	35.75	233.3	17.12	57.6	433.7	149.60	1.3	6.4	38.77	232.1	18.44	480
500	55.0	455.9	149.75	1.2	6.2	38.69	242.5	18.46	60.0	447.9	161.32	1.5	7.3	41.93	241.1	19.85	500
520	57.2	470.5	161.10	1.4	7.0	41.74	251.6	19.84	62.4	461.6	173.36	1.6	8.2	45.22	250.0	21.31	520
540	59.4	484.8	172.76	1.5	7.8	44.89	260.6	21.25	64.8	474.9	185.70	1.8	9.2	48.60	258.9	22.80	540
560	61.6	498.6	184.71	1.7	8.7	48.14	269.5	22.70	67.2	487.7	198.32	2.0	10.2	52.09	267.6	24.31	560
580	63.8	512.1	196.94	1.8	9.7	51.48	278.4	24.17	69.6	500.1	211.21	2.2	11.4	55.68	276.3	25.84	580
600	66.0	525.1	209.44	2.0	10.7	54.93	287.2	25.67	72.0	511.9	224.34	2.4	12 6	59.37	284.9	27.40	600
620	68.2	537.7	222.19	2.2	11.8	58.46	295.9	27.19	74.4	523.3	237.70	2.6	13.9	63.15	293.4	28.99	620
640	70.4	549.9	235.17	2 4	13.0	62.09	304.5	28.74	76.8	534.2	251.26	2.8	15 3	67.03	301.7	30.60	640
660	72.6	561.6	248.37	2 6	14.2	65.80	313.1	30.30	79.2	544.6	265 00	3.0	16.8	70.98	310.0	32.23	660
680	74.8	572.9	261.77	2.8	15.5	69.61	321.6	31.89	81.6	554.4	278.91	3.2	18.4	75.03	318.2	33.86	680
700	77.0	583.7	275.36	3.0	17.0	73.49	329.9	33.49	84.0	563.8	292.97	3.4	20.1	79.18	326.3	35.51	700

TABLE I.

l_1	26° Curve.								28° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
40	5.2	40.0	1.21			.39	20	.15	5.6	39.9	1.30			.33	20	.16	40
50	6.5	49.9	1.89			.47	25	.24	7.0	49.9	2.03			.51	25	.25	50
60	7.8	59.9	2.72			.68	30	.34	8.4	59.8	2.93			.74	30	.37	60
70	9.1	69.8	3.70		.01	.93	35	.46	9.8	69.8	3 99		.01	1.00	34.9	.50	70
80	10.4	79.7	4.83	.01	2	1.21	39.9	.60	11.2	79.7	5.20	.01	2	1.30	39.9	.65	80
90	11.7	89.6	6.11	1	3	1.53	44.9	.76	12.6	89.5	6.57	1	3	1.64	44.9	.82	90
100	13.0	99.5	7.54	2	5	1.89	49.9	.94	14.0	99.4	8.11	2	5	2.03	49.9	1.01	100
110	14.3	109.3	9.11	2	.6	2.28	54.9	1.14	15.4	109.2	9.80	2	8	2.46	54.8	1.22	110
120	15.6	119.1	10.83	3	9	2.71	59.8	1.35	16.8	119.0	11.65	3	.11	2.93	59.8	1.45	120
130	16.9	128.9	12.70	3	.12	3.18	64.8	1.58	18.2	128.7	13.66	4	.14	3.43	64.7	1.70	130
140	18.2	138.6	14.72	4	.15	3.69	69.7	1.84	19.6	138 4	15.83	5	.18	3.97	69.7	1.97	140
150	19.5	148.3	16.88	5	.19	4.24	74.7	2.11	21.0	148.0	18.15	6	.23	4.56	74.6	2.26	150
160	20.8	157.9	19.18	6	.24	4.82	79.6	2.40	22.4	157.6	20.62	.1	.3	5.18	79.6	2.57	160
170	22.1	167.5	21.63	8	.30	5.44	84.6	2.70	23.8	167.1	23.24	.1	.3	5.84	84.5	2.90	170
180	23.4	177.0	24.21	9	.36	6.09	89 5	3.02	25.2	176.5	26.02	.1	.4	6.54	89.4	3.24	180
190	24.7	186 5	26.94	.11	.43	6 78	94.4	3.36	26.6	185.9	28.95	.1	.5	7.29	94.3	3.61	190
200	26.0	195.9	29.81	.13	.51	7.50	99.3	3.71	28.0	195.3	32.02	.1	.6	8.07	99.2	3.99	200
210	27.3	205.3	32 81	.2	.6	8.26	104.2	4.09	29.4	204.6	35.25	.2	.7	8.89	104.1	4.39	210
220	28.6	214.6	35.95	.2	.7	9.06	109.1	4.48	30.8	213 7	38.61	.2	.8	9.75	108.9	4.81	220
230	29.9	223.8	39.23	.2	.8	9.90	113.9	4.89	32.2	222.8	42.12	.2	.9	10.64	113.8	5.24	230
240	31 2	233.0	42.64	.2	.9	10.77	118.8	5.31	33.6	231.9	45.77	.3	1.0	11.58	118.6	5.69	240
250	32.5	242.1	46.19	.2	1.0	11.67	123.7	5.75	35.0	240.8	49.56	.3	1.2	12:55	123.4	6.16	250
260	33.8	251.1	49.87	.3	1.1	12.61	128.5	6.20	36.4	249.7	53 49	.3	1.4	13.56	128.2	6.64	260
270	35.1	260.0	53.67	.3	1.3	13.59	133.3	6.67	37.8	258.5	57.55	.4	1.5	14.61	133.0	7.14	270
280	36.4	268.9	57.60	.3	1.5	14.60	138.1	7.16	39.2	267.2	61 75	.4	1.7	15.69	137.8	7.66	280
290	37.7	277.7	61.66	.4	1.6	15.65	142.9	7.66	40.6	275.8	66.07	.4	1.9	16.81	142 6	8.20	290
300	39.0	286.4	65.85	.4	1.8	16.73	147.7	8.17	42.0	284.3	70.53	.5	2.1	17.97	147 3	8.75	300
310	40.3	295.0	70.16	.5	2.0	17.85	152.4	8.70	43.4	292.7	75.12	.5	2.3	19.16	152.1	9.31	310
320	41.6	303.5	74.58	.5	2.2	19.00	157.2	9.25	44.8	301.0	79.83	.5	2.6	20 39	156.8	9.89	320
330	42 9	312.0	79 12	.6	2.4	20.18	161.9	9.81	46.2	309.2	84.66	.6	2.8	21.66	161.5	10.48	330
340	44 2	320.3	83.78	.6	2.6	21.39	166.7	10.38	47.6	317.3	89 61	.6	3.1	22.96	166 1	11.09	340
350	45.5	328.5	88.55	.7	2 9	22 64	171 4	10.97	49.0	325.2	94.68	.7	3.4	24.29	170.8	11.71	350
360	46.8	336.7	93.44	.7	3.1	23.92	176.1	11.57	50.4	333.1	99 86	.8	3.7	25.66	175.4	12 34	360
370	48.1	344.8	98.44	.7	3.4	25.23	180.8	12.18	51.8	340.9	105.16	.9	4.0	27.07	180.0	12.99	370

TABLE I.

l_1	32° Curve.								36° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
30	4.8	30	.84			.21	15	.10	5.4	30	.94			.24	15	.12	30
40	6.4	39.9	1.49			.37	20	.18	7.2	39.9	1.67			.42	20	.21	40
50	8.0	49.9	2.32		.01	.58	25	.29	9.0	49.9	2.61		.01	.65	25	.32	50
60	9.6	59.8	3.35	.01	2	.84	30	.42	10.8	59.8	3.76	.01	2	.94	30	.47	60
70	11.2	69.7	4.55	1	3	1.14	34.9	.57	12.6	69.7	5.11	1	4	1.28	34.9	.64	70
80	12.8	79 6	5.93	1	5	1.49	39.9	.74	14.4	79.5	6.67	1	6	1.67	39.9	.83	80
90	14.4	89.4	7.50	2	7	1.88	44.9	.94	16.2	89.3	8.43	2	9	2.11	44.9	1.05	90
100	16.0	99.2	9.25	2	.10	2.32	49.9	1.16	18.0	99.0	10.40	3	.13	2.61	49.8	1.30	100
110	17.6	109.0	11.18	3	.13	2.80	54.8	1.39	19.8	108 7	12.56	4	.16	3.15	54.8	1.57	110
120	19.2	118.7	13.29	4	.17	3.34	59.8	1.66	21.6	118.3	14.92	5	.22	3.75	59.7	1.86	120
130	20.8	128.3	15.58	5	.21	3.91	64.7	1.94	23.4	127.8	17.48	6	.28	4.39	64.6	2.18	130
140	22.4	137.9	18.04	7	.25	4.53	69.6	2.25	25.2	137.3	20.24	8	.35	5.09	69.5	2.52	140
150	24.0	147.4	20.68	8	.31	5.20	74.5	2.58	27.0	146.7	23.19	.10	.44	5.84	74.4	2.89	150
160	25.6	156.8	23.49	.1	.4	5.91	79.4	2.93	28.8	156.0	26.33	.1	.5	6.63	79.3	3.28	160
170	27 2	166.2	26.47	.1	.5	6.67	84.3	3.30	30.6	165.2	29.65	.1	.6	7.48	84.2	3.69	170
180	28.8	175.5	29.62	.1	.6	7.46	89.2	3.69	32.4	174.3	33.16	.2	.7	8.38	89.0	4.12	180
190	30.4	184.7	32.93	.2	.7	8.31	94.1	4.10	34.2	183.3	36.85	.2	.9	9.32	93 9	4.58	190
200	32.0	193.8	36.41	.2	.8	9.20	98.9	4.53	36.0	192.2	40.72	.2	1.0	10.31	98.7	5.06	200
210	33.6	202.9	40.05	.2	.9	10.13	103.8	4.98	37.8	201.0	44.75	.3	1.2	11.35	103.5	5 56	210
220	35.2	211.8	43.85	.3	1.1	11.10	108.6	5.45	39.6	209.7	48.97	.3	1.4	12.44	108.2	6.08	220
230	36.8	220.7	47.81	.3	1.2	12.12	113.4	5.94	41.4	218.3	53.36	.3	1.6	13.58	113.0	6.62	230
240	38.4	229.4	51.92	.3	1.4	13.19	118.2	6.45	43 2	226.7	57.92	.4	1.8	14.77	117.7	7.18	240
250	40.0	238.1	56.19	.4	1.6	14.29	123.0	6.97	45.0	235.0	62.62	.4	2.0	16.00	122.5	7.76	250
260	41.6	246.6	60.60	.4	1.8	15.43	127.7	7.52	46.8	243.2	67.48	.5	2.3	17.27	127.1	8.36	260
270	43.2	255.0	65.15	.4	2.0	16.62	132.5	8.08	48.6	251.2	72 50	.5	2.6	18.59	131.8	8.97	270
280	44.8	263.3	69.85	.5	2.3	17.84	137.2	8.66	50.4	259.1	77.67	.6	2.9	19.96	136.4	9.60	280
290	46.4	271.5	74.69	.5	2.5	19.11	141.9	9.25	52.2	266.8	82 98	.6	3.2	21.37	141.1	10.25	290
300	48.0	279.6	79.67	.6	2.8	20.42	146.5	9.86	54.0	274.4	88.43	.7	3.5	22.82	145.7	10.92	300
310	49.6	287.5	84.78	.6	3.1	21.76	151.2	10.48	55.8	281.9	94.02	.8	3.9	24.31	150.2	11.60	310
320	51.2	295.4	90.01	.7	3.4	23.15	155.8	11.12	57.6	289.2	99.73	.8	4.3	25.85	154.8	12.30	320
330	52.8	303.1	95.38	.8	3.7	24.57	160.4	11.77	59.4	296.3	105.57	.9	4.7	27.42	159.3	13.00	330
340	54.4	310.6	100.87	.8	4.1	26.04	165.0	12.44	61.2	303.2	111.54	1.0	5.1	29.04	163.8	13.72	340
350	56.0	318.0	106.48	.9	4.5	27.54	169.5	13.12	63.0	310.0	117.64	1.1	5 6	30.70	168.2	14.46	350
360	57.6	325 3	112.20	1.0	4.9	29.08	174.1	13.82	64.8	316.7	123.85	1.2	6.1	32.40	172.6	15.21	360

TABLE I.

l_1	40° Curve.								44° Curve.								l_1
	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	
30	6.0	30	1.05			.26	15	.13	6.6	29.9	1.15			.29	15	.14	30
40	8.0	39.9	1.86			.46	20	.23	8.8	39.9	2.04			.51	20	.25	40
50	10.0	49.8	2.90		.01	.73	25	.36	11.0	49.8	3.19		.01	.80	25	.40	50
60	12.0	59.7	4.17	.01	3	1.04	29.9	.52	13.2	59.7	4.59	.01	3	1.15	29.9	.57	60
70	14.0	69.6	5.68	1	5	1.42	34.9	.71	15.4	69.5	6.24	1	6	1.56	34.9	.78	70
80	16.0	79.4	7.40	2	7	1.86	39.9	.93	17.6	79.2	8.14	2	9	2.04	39.9	1.02	80
90	18.0	89.1	9.36	3	.11	2.35	44.8	1.17	19.8	88.9	10.28	3	.13	2 58	44.8	1.28	90
100	20.0	98.8	11.53	4	.15	2.89	49.8	1.44	22.0	98.5	12.66	4	.18	3.18	49.7	1.58	100
110	22.0	108.4	13.93	5	.20	3.50	54.7	1.74	24.2	108.0	15.29	6	.25	3.84	54.7	1.90	110
120	24.0	117.9	16.54	6	.27	4.16	59.7	2.06	26.4	117.5	18.15	8	.32	4.56	59.5	2.26	120
130	26.0	127.3	19.37	8	.35	4.87	64.5	2.41	28.6	126.8	21.25	.10	.41	5.35	64.5	2.65	130
140	28.0	136.7	22.42	.10	.44	5.65	69.4	2.79	30.8	136.0	24.57	.12	.52	6.21	69.3	3.06	140
150	30.0	145.9	25.67	.13	.53	6.48	74.3	3.19	33.0	145.1	28.12	.14	.65	7.11	74.2	3.50	150
160	32.0	155.1	29.13	.2	.6	7.36	79.2	3.62	35.2	154.1	31.89	.2	.8	8.08	79.0	3.97	160
170	34.0	164.1	32.79	.2	.8	8.30	84.0	4.08	37.4	162.9	35.88	.2	.9	9.10	83.8	4.46	170
180	36.0	173 0	36.65	.2	1.0	9.29	88.8	4.55	39.6	171.6	40.07	.2	1.1	10.19	88.6	4.97	180
190	38.0	181.8	40.70	.2	1.1	10.33	93.6	5.05	41.8	180.1	44.47	.3	1.3	11.33	93.3	5.51	190
200	40.0	190.4	44.95	.3	1.3	11.43	98.4	5.58	44.0	188.5	49.07	.3	1.6	12.52	98.0	6.08	200
210	42.0	199.0	49.38	.3	1.5	12.58	103.1	6.12	46.2	196.7	53.87	.4	1.8	13.78	102.7	6.67	210
220	44.0	207.4	53.99	.4	1.7	13.78	107.8	6.69	48.4	204.8	58.86	.4	2.1	15.09	107.4	7.28	220
230	46.0	215.6	58.77	.4	2.0	15 03	112.5	7.28	50.6	212.7	64.02	.5	2.4	16.46	112.0	7.91	230
240	48.0	223.7	63.73	.5	2.2	16.33	117.2	7.88	52.8	220.4	69.36	.5	2.8	17.87	116.6	8.56	240
250	50.0	231.6	68.85	.5	2.5	17.68	121.9	8.51	55.0	227.9	74.87	.6	3.1	19.34	121.2	9.23	250
260	52.0	239.4	74.14	.6	2.8	19.08	126.5	9.15	57.2	235.2	80.55	.7	3.5	20.87	125.8	9.92	260
270	54.0	247.0	79.58	.6	3.2	20.53	131.1	9.81	59.4	242.4	86.38	.7	3.9	22.45	130.3	10.62	270
280	56.0	254.4	85.18	.7	3.6	22.03	135.6	10.49	61.6	249.3	92.35	.8	4.3	24.07	134.8	11.35	280
290	58.0	261.6	90.92	.8	4.0	23 58	140.2	11.19	63.8	256.0	98.47	.9	4.8	25.74	139.2	12.08	290
300	60.0	268.7	96.79	.9	4.4	25.17	144.7	11.90	66.0	262.5	104.72	1.0	5.3	27.46	143.6	12.83	300
310	62.0	275.6	102.80	.9	4.9	26.80	149.1	12.63	68.2	268.8	111.09	1.1	5.9	29.23	147.9	13.59	310
320	64.0	282.3	108.94	1.0	5.3	28.48	153.6	13.37	70.4	274.9	117.58	1.2	6.5	31.05	152.2	14.37	320
330	66.0	288.8	115.19	1.1	5.8	30.20	158.0	14.12	72.6	280.8	124.18	1.3	7.1	32.90	156.5	15.15	330
340	68.0	295.1	121.56	1.2	6.4	31.97	162.3	14.88	74.8	286.4	130.88	1.4	7 8	34.80	160.8	15.94	340
350	70.0	301.2	128.03	1.3	7.0	33.78	166.6	15.66	77.0	291.8	137.68	1 5	8.5	36.74	165.0	16.74	350
360	72.0	307.2	134.61	1.4	7.6	35.63	170.9	16.45	79.2	297.0	144.57	1.6	9.3	38.74	169.1	17.55	360

TABLE I.

l_1	48° Curve. $I°$	x_1	y_1	e	e'	F	x'	y'	52° Curve. $I°$	x_1	y_1	e	e'	F	x'	y'	l_1
30	7.2	29.9	1.25			.32	15	.16	7.8	29.9	1.36			.34	15	0.17	30
40	9.6	39.9	2.23		.01	.56	20	.28	10.4	39.9	2.42		.01	.60	20	.30	40
50	12.0	49.8	3.48	.01	2	.87	25	.43	13.0	49.7	3.77	.01	2	.94	25	.47	50
60	14.4	59.6	5.00	1	4	1.25	29.9	.62	15.6	59.5	5.42	1	4	1.36	29.9	.68	60
70	16.8	69.4	6.80	2	7	1.70	34.9	.85	18.2	69.3	7.36	2	8	1.85	34.9	.92	70
80	19.2	79.1	8.87	3	.11	2.22	39.8	1.11	20.8	78.9	9.59	3	.12	2.41	39.8	1.20	80
90	21.6	88.7	11.19	4	.15	2.81	44.8	1.40	23.4	88.5	12.11	5	.18	3.04	44.7	1.51	90
100	24.0	98.2	13.79	5	.22	3.47	49.7	1.72	26.0	97.9	14.90	6	.26	3.75	49.6	1.86	100
110	26.4	107.7	16.64	7	.29	4.19	54.6	2.07	28.6	107.3	17.98	.1	.3	4.53	54.5	2.24	110
120	28.8	117.0	19.74	9	.38	4.98	59.5	2.46	31.2	116.5	21.32	.1	.4	5.38	59.4	2.65	120
130	31.2	126.2	23 10	.11	.49	5.83	64.3	2.87	33.8	125.5	24.93	.1	.6	6.31	64.2	3.10	130
140	33.6	135.2	26.70	.14	.62	6.75	69.2	3.32	36.4	134.4	28.80	.2	.7	7.30	69.1	3.58	140
150	36.0	144.2	30.54	.17	.77	7.74	74.0	3.79	39.0	143.2	32.92	.2	.9	8.37	73.8	4.09	150
160	38.4	153 0	34.61	.2	.9	8.79	78.8	4.30	41.6	151.8	37.29	.3	1.1	9.50	78.6	4.62	160
170	40.8	161.6	38.91	.2	1.1	9 90	83.6	4.83	44.2	160.1	41 89	.3	1.3	10.70	83.3	5.19	170
180	43 2	170.0	43.43	.3	1.3	11.07	88.3	5.39	46.8	168.3	46.72	.4	1.6	11.96	88.0	5.79	180
	56° Curve.								60° Curve.								
30	8.4	29.9	1.46			.37	15	.18	9.0	29.9	1.57		.01	.39	15	.20	30
40	11.2	39.8	2.60		.01	.65	20	.32	12.0	39.8	2.78	.01	2	.70	20	.35	40
50	14.0	49.7	4.06	.01	2	1.02	24.9	.51	15.0	49.7	4.34	1	4	1.09	24.9	.54	50
60	16.8	59.5	5.83	2	5	1.46	29.9	.73	18.0	59.4	6.24	2	7	1.57	29.9	.78	60
70	19.6	69.2	7.92	3	9	1.99	34.9	.99	21.0	69.1	8.47	3	.12	2.13	34.8	1.06	70
80	22.4	78.8	10.31	4	.15	2.59	39.8	1.29	24.0	78.6	11.03	4	.18	2.77	39.8	1.37	80
90	25.2	88 3	13.01	5	21	3.27	44.7	1.62	27.0	88.0	13.91	6	.25	3.50	44.7	1.73	90
100	28.0	97.6	16.01	6	.30	4.03	49.6	1.99	30.0	97.3	17.12	8	.35	4.32	49.5	2.13	100
110	30.8	106.9	19.31	.1	.4	4.87	54.5	2.40	33.0	106.4	20.62	.1	.5	5.21	54.4	2.57	110
120	33.6	116.0	22.89	.1	.5	5.79	59.3	2.85	36.0	115.3	24.43	.1	.6	6.19	59.2	3.04	120
130	36 4	124.9	26.75	.2	.7	6.78	64.1	3.32	39.0	124.1	28.53	.2	.8	7.25	64.0	3.54	130
140	39.2	133.6	30.88	.2	.9	7.85	68.9	3.83	42.0	132.7	32.92	.2	1.0	8.38	68.8	4.08	140
150	42.0	142.2	35.27	.2	1.2	8.98	73.7	4.37	45.0	141.0	37.58	.2	1.2	9.60	73.5	4.65	150
160	44.8	150 5	39.92	.3	1.3	10.20	78.4	4.95	48.0	149.1	42.49	.3	1.5	10.89	78.2	5.26	160
170	47.6	158.6	44.81	.3	1.5	11.48	83.1	5.55	51.0	157.0	47.65	.4	1.8	12.26	82.8	5.89	170
180	50.4	166.6	49.93	.4	1.8	12.83	87.7	6.17	54.0	164.7	53.06	.4	2.1	13.69	87.4	6.55	180

TABLE I.

l_1	64° Curve. $I°$	x_1	y_1	e	e'	F	x'	y'	68° Curve. $I°$	x_1	y_1	e	e'	F	x'	y'	l_1
30	9.6	29.9	1.67		.01	.42	15	21	10.2	29.9	1.78			.44	15	.22	30
40	12.8	39.8	2.97	.01	2	.74	20	.37	13.6	39.8	3.15	.01	.01	.79	20	.40	40
50	16.0	49.6	4.63	1	4	1.16	24.9	.58	17.0	49.6	4.91	1	4	1.23	24.9	.61	50
60	19.2	59.3	6.65	2	7	1.67	29.9	.83	20.4	59.2	7.06	2	7	1.77	29.9	.87	60
70	22.4	68.9	9.02	3	.12	2.27	34.8	1.12	23.8	68.8	9.58	4	.12	2.41	34.8	1.18	70
80	25.6	78.4	11.75	5	.18	2.96	39.7	1.46	27.2	78.2	12.46	5	.19	3.14	39.7	1.55	80
90	28.8	87.7	14.81	7	.27	3.73	44.6	1.84	30.6	87.5	15.70	8	.29	3.96	44.6	1.95	90
100	32.0	96.9	18.21	.10	.38	4.60	49.5	2.26	34.0	96.5	19 29	.11	.42	4.88	49.4	2.39	100
110	35.2	105.9	21.93	.1	.5	5.55	54.3	2.72	37.4	105.4	23.22	.1	.6	5.89	54.2	2.88	110
120	38.4	114.7	25.96	.2	.7	6.59	59.1	3.22	40.8	114.1	27.47	.2	.8	6.99	59.0	3.40	120
130	41.6	123.3	30.30	.2	.9	7.71	63.9	3.76	44.2	122.5	32.03	.2	1.0	8.18	63.7	3.96	130
140	44.8	131.6	34.93	.2	1.1	8.92	68.6	4.33	47.6	130.6	36.89	.3	1.2	9.45	68.4	4.55	140
150	48.0	139.8	39.83	.3	1.3	10.21	73.3	4.93	51.0	138.5	42.05	.3	1.5	10.81	73.1	5.19	150
160	51.2	147.7	45.01	.4	1.6	11.58	77.9	5 56	54.4	146.2	47.47	.4	1.8	12.26	77.7	5.86	160
170	54.4	155.3	50.44	.4	2.0	13.02	82.5	6.22	57.8	153.5	53.14	.5	2.2	13.78	82.2	6.55	170
180	57.6	162.6	56.10	.5	2 4	14.54	87.0	6.91	61.2	160.5	59.05	.5	2.7	15.38	86.7	7.27	180
	72° Curve.								76° Curve.								
10	3.6	10	.21			.05	5	.03	3.8	10	.22			.06	5	.03	10
20	7.2	20	.84			.21	10	.10	7.6	20	.88		.01	.22	10	.11	20
30	10.8	29.9	1.88		.01	.47	15	.23	11.4	29.9	1.98		2	.49	15	.24	30
40	14.4	39.7	3.34	.01	2	.84	20	.41	15.2	39.7	3.52	.01	3	.88	20	.43	40
50	18.0	49.5	5.20	2	5	1.31	24.9	.65	19.0	49.4	5.48	.02	6	1.38	24.9	.68	50
60	21.6	59.2	7.46	3	9	1.88	29.8	.93	22.8	59.1	7.87	.03	.11	1.98	29.8	.98	60
70	25.2	68.7	10.12	4	.14	2.55	34.7	1.26	26.6	68.5	10.67	.05	.17	2.69	34.7	1.33	70
80	28.8	78.0	13.16	5	.23	3.32	39.6	1.64	30.4	77.8	13.87	.07	.27	3.50	39.6	1.72	80
90	32.4	87.2	16.58	9	.34	4.19	44.5	2.06	34.2	86.8	17.46	.10	.40	4.42	44.5	2.16	90
100	36.0	96.1	20.36	.12	.48	5.15	49.3	2.53	38.0	95.7	21.42	.13	.56	5.44	49.3	2.65	100
110	39.6	104.8	24.49	.2	.6	6.22	54.1	3.04	41.8	104.3	25.75	.2	.7	6.56	54.0	3.19	110
120	43.2	113.3	28.96	.2	.9	7.38	58.8	3.59	45.6	112.6	30.42	.2	1.0	7.78	58.7	3.77	120
130	46.8	121.6	33.74	.2	1.1	8.63	63.5	4.18	49.4	120.7	35.42	.3	1.3	9.09	63.4	4.38	130
140	50.4	129.5	38.83	.3	1.4	9.98	68.2	4.80	53.2	128.4	40.73	.3	1.6	10.50	68.0	5.03	140
150	54.0	137.2	44.21	.3	1.7	11.41	72.8	5.46	57.0	135.8	46.33	.4	2.0	12.00	72.6	5.71	150
160	57.6	144.6	49.87	.4	2.1	12.93	77.4	6.15	60.8	142.9	52.20	.5	2.4	13.59	77.1	6.41	160

TABLE I.

80° Curve.

l_1	$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
15	6.0	14.99	.52			.13	7.50	.06	15
20	8.0	19.96	.93			.23	9.99	.11	20
25	10.0	24.92	1.45		.01	.36	12.49	.18	25
30	12.0	29.87	2.09		2	.52	14.98	.26	30
35	14.0	34.79	2.84	.01	3	.71	17.46	.35	35
40	16.0	39.69	3.70	1	4	.93	19.95	.46	40
45	18.0	44.56	4.68	1	6	1.17	22.43	.58	45
50	20.0	49.39	5.77	2	8	1.45	24.90	.72	50
55	22.0	54.19	6.97	2	.10	1.75	27.36	.87	55
60	24.0	58.96	8.27	3	.14	2.08	29.82	1.03	60
65	26.0	63.68	9.69	4	.18	2.44	32.27	1.21	65
70	28.0	68.35	11.21	5	.22	2.83	34.72	1.39	70
75	30.0	72.97	12.84	6	.27	3.24	37.16	1.60	75
80	32.0	77.54	14.56	8	.33	3.68	39.59	1.81	80
85	34.0	82.05	16.39	.10	.40	4.15	42.00	2.04	85
90	36.0	86.51	18.32	.11	.46	4.64	44.42	2.28	90

88° Curve.

$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
6.6	14.98	.57			.14	7.50	.07	15
8.8	19.95	1.02			.25	9.99	.13	20
11.0	24.91	1.59		.01	.40	12.49	.20	25
13.2	29.84	2.29		2	.57	14.98	.29	30
15.4	34.75	3.12	.01	3	.78	17.46	.39	35
17.6	39.62	4.08	1	5	1.02	19.94	.51	40
19.8	44.46	5.15	1	7	1.29	22.41	.64	45
22.0	49.27	6.33	2	.10	1.59	24.87	.79	50
24.2	54.03	7.64	3	.13	1.92	27.34	.95	55
26.4	58.74	9.07	4	.17	2.28	29.79	1.13	60
28.6	63.40	10.62	5	.22	2.68	32.24	1.32	65
30.8	68.01	12.28	6	.26	3.10	34.67	1.53	70
33.0	72.56	14.06	7	.31	3.55	37.10	1.75	75
35.2	77.05	15.95	9	.38	4.04	39.51	1.98	80
37.4	81.46	17.94	.11	.46	4.55	41.91	2.23	85
39.6	85.82	20.04	.12	.55	5.09	44.31	2.49	90

96° Curve.

$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
7.2	14.98	.63			.16	7.50	.08	15
9.6	19.95	1.11		.01	.28	9.99	.14	20
12.0	24.89	1.74		1	.44	12.47	.22	25
14.4	29.81	2.50	.01	2	.63	14.96	.31	30
16.8	34.70	3.40	1	4	.85	17.45	.42	35
19.2	39.55	4.43	1	6	1.11	19.93	.55	40
21.6	44.36	5.60	2	8	1.40	22.39	.70	45
24.0	49.12	6.89	3	.11	1.73	24.85	.86	50
26.4	53.85	8.32	4	.15	2.09	27.30	1.04	55
28.8	58.50	9.87	5	.20	2.49	29.75	1.23	60
31.2	63.10	11.55	6	.26	2.92	32.18	1.44	65
33.6	67.63	13.35	7	.31	3.38	34.60	1.66	70
36.0	72.10	15.27	9	.39	3.87	37.01	1.90	75
38.4	76.49	17.31	.10	.47	4.39	39.40	2.15	80
40.8	80.79	19.46	.12	.56	4.95	41.79	2.41	85
43.2	85.03	21.72	.15	.70	5.54	44.16	2.69	90

104° Curve.

$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
7.8	14.97	.68			.17	7.50	.08	15
10.4	19.94	1.21		.01	.30	9.99	.15	20
13.0	24.88	1.88		1	.47	12.47	.23	25
15.6	29.77	2.71	.01	3	.68	14.96	.34	30
18.2	34.65	3.68	1	4	.92	17.44	.46	35
20.8	39.48	4.80	2	7	1.20	19.91	.60	40
23.4	44.26	6.05	2	9	1.52	22.37	.75	45
26.0	48.98	7.45	3	.13	1.88	24.82	.93	50
28.6	53.65	8.99	4	.17	2.27	27.27	1.12	55
31.2	58.25	10.66	5	.23	2.69	29.70	1.33	60
33.8	62.77	12.46	6	.29	3.15	32.13	1.55	65
36.4	67.22	14.40	9	.36	3.65	34.54	1.79	70
39.0	71.60	16.48	.10	.45	4.18	36.93	2.04	75
41.6	75.89	18.66	.12	.55	4.75	39.30	2.31	80
44.2	80.08	20.95	.15	.66	5.35	41.66	2.60	85
46.8	84.19	23.36	.18	.79	5.98	44.02	2.90	90

TABLE I.

112° Curve.

l_1	I^c	x_1	y_1	e	e'	F	x'	y'	l_1
10	5.6	9.99	.32			.08	5.00	.04	10
15	8.4	14.96	.73			.18	7.50	.09	15
20	11.2	19.92	1.30		.01	.32	9.99	.16	20
25	14.0	24 85	2.03		2	.51	12 48	.25	25
30	16.8	29.74	2.91	.01	3	.73	14.96	.36	30
35	19.6	34.59	3.96	1	5	.99	17.43	.49	35
40	22.4	39.39	5.16	2	8	1.30	19 90	.64	40
45	25.2	44.14	6.51	3	.11	1.64	22.35	.81	45
50	28.0	48.82	8.01	4	.15	2 02	24.80	1.00	50
55	30.8	53.44	9.65	5	.20	2 44	27 24	1.20	55
60	33 6	57.98	11.44	6	.26	2.90	29 66	1.42	60
65	36.4	62.43	13.37	8	.33	3.39	32 06	1.66	65
70	39.2	66.80	15.44	.10	.42	3.92	34.46	1 92	70
75	42 0	71.08	17.63	.11	.52	4.49	36.84	2.19	75
80	44.8	75.25	19.96	.14	.64	5 10	39.20	2.47	80
85	47.6	79.32	22.40	.17	.76	5.74	41.54	2.77	85

120° Curve.

l_1	$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
6.0	9.99	.35			.09	5.00	.04	10	
9.0	14.96	.78			.20	7.49	.10	15	
12.0	19.91	1.39		.01	.35	9.98	.17	20	
15.0	24.82	2.17	.01	2	.54	12.47	.27	25	
18 0	29.70	3.12	1	3	.78	14.95	.39	30	
21.0	34.53	4.24	1	6	1.06	17.42	.53	35	
24.0	39.30	5.52	2	9	1.39	19.88	69	40	
27.0	44.00	6.96	3	.13	1.75	22.33	.87	45	
30.0	48.64	8.56	4	.18	2.16	24.77	1.07	50	
33.0	53.20	10.31	5	.23	2.61	27.20	1.29	55	
36.0	57.68	12.22	7	.31	3.10	29 61	1.52	60	
39.0	62.06	14.27	8	.40	3.62	32.00	1.77	65	
42.0	66.33	16.46	.11	.50	4.19	34.38	2.04	70	
45.0	70.50	18.79	.13	.61	4.80	36.74	2.33	75	
48.0	74.56	21.25	.16	.74	5.43	39.08	2.63	80	
51.0	78.51	23 83	.19	.89	6.13	41.39	2.94	85	

128° Curve.

10	6.4	9.99	.37			.09	5.00	.05	10
15	9.6	14.96	.84			.21	7.49	.10	15
20	12.8	19.90	1.48		.01	.37	9.98	.18	20
25	16.0	24.81	2.31	.01	2	.58	12.46	.29	25
30	19.2	29.66	3.32	1	4	.83	14.94	.42	30
35	22.4	34.47	4.51	2	6	1.13	17.41	.56	35
40	25.6	39.21	5.87	2	.10	1.48	19.86	.73	40
45	28.8	43.87	7.40	4	.14	1.87	22.31	.92	45
50	32.0	48.46	9.10	5	.20	2.30	24.74	1.13	50
55	35.2	52.96	10.96	6	.27	2.78	27.16	1.36	55
60	38.4	57.36	12.98	7	.35	3.30	29.56	1.61	60
65	41.6	61.66	15.15	.10	.45	3.86	31.94	1.88	65
70	44.8	65.84	17.46	.12	.56	4.46	34.40	2.16	70
75	48.0	69.90	19.92	.15	.70	5.10	36.64	2.46	75
80	51.2	73.85	22.50	.17	.85	5.79	38.96	2.78	80
85	54.4	77.65	25.22	.21	1.02	6.51	41.25	3.11	85

136° Curve.

6.8	9.99	.40			.10	5.00	.05	10	
10.2	14.96	.89			.22	7.49	.11	15	
13.6	19.89	1.58		.01	.39	9.98	.20	20	
17.0	24.78	2.46	.01	2	.62	12.46	.31	25	
20.4	29.62	3 53	1	4	.89	14.93	.44	30	
23.8	34.40	4.79	2	7	1.20	17.39	.60	35	
27.2	39.11	6.23	3	.11	1.56	19.84	.78	40	
30.6	43.73	7.85	4	.16	1.98	22.29	.98	45	
34.0	48.27	9.64	6	.22	2.44	24.71	1.20	50	
37.4	52.71	11.60	7	.31	2.94	27.11	1.44	55	
40.8	57.03	13.73	9	.40	3.50	29.49	1.70	60	
44.2	61.24	16.02	.11	.50	4.09	31.86	1.98	65	
47.6	65.32	18.45	.14	.63	4.73	34.21	2.28	70	
51.0	69.27	21.02	.17	.79	5.41	36.53	2.60	75	
54.4	73.08	23.73	.19	.96	6.14	38.83	2.93	80	
57.8	76.74	26.57	.24	1.16	6.90	41.10	3.28	85	

TABLE 1*a*.

l_1	$I°$	x_1	y_1	e	e'	F	x'	y'
			RADIUS = 110.					
30	7.81	29.94	1.36		.01	.34	14.99	.17
40	10.42	39.87	2.42		2	.60	19.98	.30
50	13.02	49.74	3.77	.01	3	.94	24.96	.47
60	15.63	59.55	5.43	1	7	1.36	29.92	.67
70	18.23	69.29	7.37	2	.10	1.85	34.88	.92
80	20.83	78.95	9.60	3	.14	2.41	39.83	1.20
90	23.44	88.51	12.13	5	.19	3.05	44.76	1.51
100	26.04	97.95	14.93	6	.29	3.76	49.65	1.86
110	28.65	107.28	18.00	8	.38	4.54	54.54	2.24
120	31.25	116.48	21.36	.11	.47	5.40	59.42	2.64
			RADIUS = 102.					
30	8.43	29.93	1.47		.01	.37	14.99	.18
40	11.23	39.85	2.61		2	.65	19.98	.33
50	14.04	49.70	4.07	.01	4	1.02	24.95	.51
60	16.85	59.48	5.85	2	7	1.46	29.91	.73
70	19.66	69.18	7.94	3	.11	1.99	34.86	.99
80	22.47	78.78	10.34	4	.16	2.60	39.80	1.29
90	25.28	88.26	13.05	5	.24	3.28	44.71	1.63
100	28.09	97.62	16.06	7	.33	4.04	49.60	1.99
110	30.89	106.85	19.36	.10	.42	4.88	54.48	2.40
120	33.70	115.91	22.95	.12	.54	5.80	59.34	2.85
			RADIUS = 94.					
30	9.14	29.92	1.59		.01	.40	14.99	.20
40	12.19	39.82	2.83	.01	2	.71	19.97	.35
50	15.24	49.65	4.41	1	5	1.11	24.94	.55
60	18.29	59.39	6.34	2	9	1.59	29.89	.79
70	21.33	69.04	8.60	3	.13	2.16	34.84	1.07
80	24.38	78.57	11.20	4	.20	2.81	39.76	1.39
90	27.43	87.96	14.13	6	.29	3.56	44.65	1.76
100	30.48	97.20	17.38	9	.39	4.40	49.52	2.17
110	33.52	106.29	20.93	.11	.50	5.30	54.38	2.61
120	36.57	115.20	24.80	.14	.66	6.30	59.20	3.08

$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
		RADIUS = 106.						
8.11	29.94	1.41		.01	.35	14.99	.18	30
10.81	39.86	2.51		2	.63	19.98	.31	40
13.51	49.72	3.91	.01	3	.98	24.96	.49	50
16.22	59.52	5.63	1	7	1.41	29.92	.70	60
18.92	69.24	7.65	2	.10	1.92	34.88	.96	70
21.62	78.87	9.96	4	.14	2.51	39.82	1.25	80
24.32	88.39	12.57	5	.21	3.17	44.74	1.57	90
27.03	97.80	15.48	7	.30	3.90	49.63	1.92	100
29.73	107.08	18.68	9	.40	4.72	54.51	2.31	110
32.43	116.21	22.14	.12	.51	5.61	59.37	2.75	120
		RADIUS = 98.						
8.77	29.93	1.53		.01	.38	14.99	.19	30
11.69	39.83	2.71		3	.68	19.97	.34	40
14.62	49.68	4.23	.01	5	1.06	24.94	.53	50
17.54	59.44	6.08	2	8	1.53	29.90	.76	60
20.46	69.11	8.26	3	.12	2.07	34.85	1.03	70
23.39	78.68	10.76	4	.18	2.70	39.78	1.34	80
26.31	88.12	13.57	6	.25	3.42	44.69	1.69	90
29.23	97.43	16.69	8	.35	4.21	49.57	2.07	100
32.15	106.59	20.12	.10	.46	5.09	54.44	2.49	110
35.08	115.59	23.85	.13	.60	6.05	59.27	2.96	120
		RADIUS = 90.						
9.55	29.92	1.66		.01	.41	14.99	.21	30
12.73	39.80	2.95	.01	2	.74	19.97	.37	40
15.92	49.62	4.61	1	6	1.15	24.93	.58	50
19.10	59.34	6.62	2	9	1.66	29.89	.83	60
22.28	68.95	8.98	3	.14	2.26	34.83	1.12	70
25.46	78.44	11.68	5	.20	2.93	39.75	1.46	80
28.65	87.78	14.73	7	.30	3.70	44.63	1.84	90
31.83	96.96	18.11	9	.42	4.57	49.49	2.26	100
35.01	105.96	21.81	.12	.54	5.53	54.33	2.72	110
38.20	114.77	25.83	.16	.72	6.57	59.12	3.21	120

TABLE I*a*.

l_1	$I°$	x_1	y_1	e	e'	F	x'	y'
			Radius = 86.					
20	6.66	19.97	.77			.19	10.	.10
30	9.99	29.91	1.74		.01	.43	14.99	.22
40	13.32	39.78	3.09	.01	2	.77	19.97	.39
50	16.66	49.58	4.82	1	6	1.21	24.93	.60
60	19.99	59.27	6.92	2	.11	1.74	29.87	.86
70	23.32	68.85	9.38	4	.16	2.36	34.80	1.17
80	26.65	78.29	12.21	5	.24	3.07	39.71	1.51
90	29.98	87.57	15.39	7	.34	3.88	44.59	1.90
100	33.31	96.67	18.91	.10	.46	4.79	49.44	2.34
110	36.64	105.59	22.77	.13	.61	5.79	54.26	2.82
			Radius = 78.					
20	7.34	19.97	.85		.01	.21	9.99	.11
30	11.02	29.89	1.92		1	.48	14.99	.24
40	14.69	39.74	3.40	.01	3	.85	19.96	.42
50	18.36	49.49	5.30	1	7	1.33	24.92	.66
60	22.04	59.12	7.61	3	.12	1.91	29.85	.95
70	25.71	68.60	10.32	4	.20	2.60	34.76	1.29
80	29.38	77.92	13.42	6	.29	3.39	39.65	1.67
90	33.05	87.05	16.90	9	.40	4.28	44.51	2.10
100	36.73	95.97	20.75	.12	.56	5.26	49.32	2.57
110	40.40	104.66	24.95	.16	.73	6.35	54.11	3.08
			Radius = 70.					
20	8.19	19.96	.95		.01	.24	9.99	.12
30	12.28	29.86	2.14		3	.54	14.97	.27
40	16.37	39.67	3.79	.01	4	.95	19.95	.47
50	20.46	49.37	5.90	2	8	1.48	24.90	.73
60	24.55	58.91	8.46	3	.14	2.13	29.83	1.05
70	28.65	68.27	11.46	5	.23	2.89	34.72	1.42
80	32.74	77.43	14.89	8	.35	3.77	39.57	1.85
90	36.83	86.35	18.72	.11	.50	4.75	44.39	2.33
100	40.92	95.02	22.95	.15	.68	5.85	49.17	2.85
110	45.02	103.40	27.56	.19	.91	7.05	53.90	3.42

$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
		Radius = 82.						
6.99	19.97	.81		.01	.20	9.99	.10	20
10.48	29.90	1.82		1	.45	14.99	.23	30
13.97	39.76	3.24	.01	2	.81	19.97	.41	40
17.47	49.54	5.05	1	6	1.26	24.93	.63	50
20.96	59.20	7.25	2	.11	1.81	29.87	.91	60
24.46	68.73	9.83	4	.18	2.47	34.78	1.23	70
27.95	78.11	12.79	6	.26	3.23	39.68	1.59	80
31.44	87.32	16.11	8	.37	4.08	44.55	2.00	90
34.94	96.34	19.79	.11	.51	5.02	49.38	2.46	100
38.43	105.15	23.81	.15	.67	6.04	54.18	2.95	110
		Radius = 74.						
7.74	19.96	.90		.01	.23	9.99	.11	20
11.61	29.88	2.02		1	.51	14.99	.25	30
15.49	39.71	3.59	.01	3	.90	19.96	.45	40
19.36	49.43	5.58	2	8	1.40	24.90	.70	50
23.23	59.02	8.01	3	.14	2.01	29.83	1.00	60
27.10	68.45	10.86	5	.21	2.74	34.74	1.36	70
30.97	77.69	14.12	7	.32	3.57	39.61	1.76	80
34.84	86.73	17.77	.10	.45	4.51	44.45	2.20	90
38.71	95.53	21.80	.13	.62	5.55	49.25	2.69	100
42.58	104.08	26.19	.17	.82	6.68	54.01	3.24	110
		Radius = 66.						
8.68	19.96	1.01		.01	.25	9.99	.13	20
13.02	29.84	2.26		3	.56	14.97	.29	30
17.36	39.63	4.01	.01	5	1.00	19.94	.50	40
21.70	49.29	6.25	2	9	1.57	24.89	.78	50
26.04	58.77	8.95	4	.16	2.26	29.80	1.12	60
30.39	68.06	12.12	6	.27	3.06	34.67	1.51	70
34.73	77.11	15.74	9	.40	3.98	39.51	1.95	80
39.07	85.90	19.79	.12	.58	5.02	44.30	2.44	90
43.41	94.41	24.24	.16	.78	6.18	49.05	2.98	100
47.75	102.59	29.07	.22	1.04	7.46	53.74	3.60	110

TABLE I*a*.

l_1	$I°$	x_1	y_1	e	e'	F	x'	y'	$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
	RADIUS = 62.								RADIUS = 58.								
20	9.24	19.95	1.07		.01	.27	9.99	.13	9.88	19.94	1.15		.01	.29	9.99	.14	20
25	11.55	24.90	1.67		1	.42	12.49	.21	12.35	24.88	1.79		2	.45	12.48	.23	25
30	13.86	29.82	2.41	.01	2	.60	14.97	.30	14.82	29.80	2.57	.01	2	.64	14.97	.32	30
35	16.17	34.72	3.27	1	3	.82	17.46	.41	17.29	34.70	3.50	1	3	.87	17.46	.44	35
40	18.48	39.59	4.27	1	6	1.07	19.93	.54	19.76	39.54	4.56	2	5	1.14	19.93	.57	40
45	20.79	44.41	5.39	2	8	1.35	22.40	.68	22.23	44.33	5.76	2	9	1.45	22.39	.72	45
50	23.10	49.19	6.64	2	.12	1.67	24.86	.83	24.70	49.08	7.09	3	.13	1.78	24.84	.88	50
55	25.41	53.93	8.02	3	.15	2.02	27.32	1.00	27.17	53.78	8.55	4	.16	2.15	27.30	1.06	55
60	27.72	58.61	9.51	4	.19	2.40	29.77	1.19	29.64	58.41	10.15	5	.21	2.56	29.74	1.26	60
65	30.03	63.24	11.13	5	.24	2.80	32.21	1.39	32.10	62.99	11.87	6	.27	3.00	32.17	1.47	65
	RADIUS = 54.								RADIUS = 50.								
20	10.61	19.93	1.23		.01	.31	9.99	.15	11.46	19.92	1.33		.01	.33	9.99	.17	20
25	13.26	24.86	1.92		2	.48	12.48	.23	14.32	24.84	2.07		2	.52	12.48	.26	25
30	15.92	29.76	2.76	.01	3	.69	14.96	.34	17.19	29.73	2.98	.01	3	.75	14.96	.38	30
35	18.57	34.63	3.75	1	5	.94	17.44	.47	20.05	34.57	4.05	1	6	1.02	17.43	.51	35
40	21.22	39.45	4.89	2	7	1.23	19.91	.61	22.92	39.36	5.27	2	9	1.33	19.89	.66	40
45	23.87	44.23	6.17	2	.11	1.55	22.37	.77	25.78	44.09	6.65	3	.13	1.67	22.34	.83	45
50	26.53	48.94	7.60	3	.15	1.91	24.82	.95	28.65	48.76	8.19	4	.17	2.07	24.79	1.01	50
55	29.18	53.59	9.17	4	.20	2.31	27.26	1.14	31.51	53.36	9.87	5	.22	2.50	27.23	1.22	55
60	31.83	58.17	10.87	5	.26	2.75	29.69	1.35	34.37	57.88	11.69	6	.28	2.96	29.65	1.45	60
65	34.48	62.69	12.70	7	.31	3.22	32.12	1.58	37.24	62.31	13.66	8	.37	3.46	32.05	1.70	65
	RADIUS = 48.								RADIUS = 46.								
20	11.94	19.91	1.38		.01	.35	9.99	.17	12.46	19.91	1.44		.01	.36	9.99	.18	20
25	14.92	24.83	2.16	.01	2	.54	12.47	.27	15.57	24.82	2.25	.01	2	.56	12.47	.28	25
30	17.90	29.71	3.10	1	3	.78	14.96	.39	18.68	29.68	3.24	1	4	.82	14.95	.40	30
35	20.89	34.54	4.21	1	6	1.06	17.43	.53	21.80	34.50	4.39	2	6	1.11	17.42	.55	35
40	23.87	39.31	5.49	2	9	1.38	19.89	.69	24.91	39.25	5.72	2	.10	1.44	19.88	.72	40
45	26.86	44.02	6.92	3	.14	1.74	22.33	.87	28.03	43.93	7.21	3	.15	1.81	22.32	.90	45
50	29.84	48.66	8.51	4	.19	2.15	24.77	1.06	31.14	48.54	8.87	4	.21	2.23	24.75	1.10	50
55	32.83	53.22	10.26	5	.26	2.60	27.19	1.27	34.25	53.07	10.68	6	.27	2.70	27.17	1.33	55
60	35.81	57.70	12.16	7	.32	3.09	29.61	1.51	37.37	57.50	12.65	7	.35	3.21	29.58	1.57	60
65	38.79	62.08	14.20	9	.40	3.61	32.01	1.76	40.48	61.83	14.77	.10	.43	3.76	31.97	1.83	65

TABLE I*a*.

l_1	$I°$	x_1	y_1	e	e'	F	x'	y'
			Radius = 44.					
15	9.77	14.96	.85			.21	7.50	.11
20	13.02	19.90	1.51		.01	.38	9.99	.19
25	16.28	24.80	2.35	.01	3	.59	12.46	.29
30	19.53	29.65	3.38	1	5	.85	14.94	.42
35	22.79	34.45	4.59	2	7	1.15	17.41	.57
40	26.04	39.18	5.97	3	.11	1.50	19.86	.74
45	29.30	43.84	7.53	4	.15	1.90	22.31	.94
50	32.55	48.41	9.25	5	.21	2.34	24.74	1.15
55	35.81	52.89	11.14	6	.29	2.82	27.15	1.39
60	39.07	57.27	13.19	7	.39	3.35	29.54	1.64
			Radius = 40.					
15	10.74	14.95	.93		.01	.23	7.49	.12
20	14.32	19.87	1.66		2	.42	9.98	.21
25	17.90	24.76	2.58	.01	3	.65	12.46	.32
30	21.49	29.58	3.71	1	6	.93	14.93	.46
35	25.07	34.34	5.04	2	9	1.27	17.39	.63
40	28.65	39.01	6.55	3	.14	1.66	19.83	.82
45	32.23	43.60	8.25	4	.20	2.08	22.26	1.03
50	35.81	48.08	10.13	6	.26	2.57	24.68	1.26
55	39.39	52.46	12.19	8	.34	3.10	27.08	1.51
60	42.97	56.71	14.41	.10	.45	3.68	29.45	1.79
			Radius = 36.					
15	11.94	14.93	1.04		.02	.26	7.48	.13
20	15.92	19.85	1.84		2	.46	9.98	.23
25	19.90	24.70	2.87	.01	4	.72	12.45	.36
30	23.87	29.48	4.11	2	7	1.03	14.91	.51
35	27.85	34.18	5.58	3	.11	1.41	17.36	.69
40	31.83	38.78	7.25	4	.17	1.84	19.79	.90
45	35.81	43.27	9.12	5	.24	2.32	22.21	1.13
50	39.79	47.64	11.18	7	.33	2.85	24.60	1.38
55	43.77	51.88	13.43	9	.43	3.43	26.98	1.66
60	47.75	55.96	15.86	.12	.56	4.07	29.32	1.96

$I°$	x_1	y_1	e	e'	F	x'	y'	l_1
		Radius = 42.						
10.23	14.95	.89		.01	.22	7.49	.11	15
13.64	19.89	1.58		2	.40	9.98	.20	20
17.05	24.78	2.46	.01	2	.61	12.47	.30	25
20.46	29.62	3.54	1	5	.88	14.94	.44	30
23.87	34.39	4.80	2	8	1.21	17.40	.60	35
27.28	39.10	6.25	3	.12	1.58	19.85	.78	40
30.69	43.73	7.87	4	.17	2.00	22.29	.98	45
34.10	48.26	9.67	5	.24	2.46	24.71	1.20	50
37.51	52.69	11.64	7	.31	2.96	27.12	1.44	55
40.92	57.01	13.77	8	.42	3.51	29.50	1.71	60
		Radius = 38.						
11.31	14.94	.98		.01	.24	7.49	.12	15
15.08	19.86	1.75		2	.44	9.98	.22	20
18.85	24.73	2.72	.01	4	.68	12.45	.34	25
22.62	29.54	3.90	1	6	.98	14.93	.49	30
26.39	34.27	5.29	2	.10	1.33	17.38	.66	35
30.16	38.91	6.88	3	.15	1.73	19.82	.86	40
33.92	43.45	8.66	5	.20	2.19	22.25	1.08	45
37.69	47.88	10.63	6	.29	2.70	24.65	1.32	50
41.46	52.19	12.78	8	.39	3.25	27.03	1.59	55
45.23	56.37	15.10	.11	.50	3.86	29.39	1.87	60
		Radius = 35.						
12.28	14.93	1.07		.02	.27	7.48	.13	15
16.37	19.84	1.89	.01	1	.47	9.98	.24	20
20.46	24.68	2.95	1	4	.74	12.45	.37	25
24.55	29.45	4.23	2	7	1.06	14.91	.53	30
28.65	34.13	5.73	3	.12	1.44	17.35	.71	35
32.74	38.71	7.44	4	.18	1.88	19.78	.92	40
36.83	43.18	9.36	5	.25	2.38	22.20	1.15	45
40.92	47.52	11.48	7	.33	2.93	24.60	1.41	50
45.02	51.71	13.78	.10	.46	3.53	26.94	1.70	55
49.11	55.74	16.26	.12	.61	4.17	29.27	2.00	60

TABLE IIa. COÖRDINATES. $\phi = I\frac{x^2}{l_1^2} + FI\frac{x^2}{l_1^2}$. $y = Gx$.

$\phi°$	G .1°	F .1°	$\phi°$	G .1°	F .1°	$\phi°$	G .1°	F .1°	$\phi°$	G .1°	F .1°
.5	0.00291 582	0.00002 8	.5	0.09636 586	0.01475 196	.5	0.17888 596	0.05837 398	.5	0.26952 614	0.13476 636
1	582 580	6 14	16	9329 586	1573 202	31	.18186 596	6036 406	46	.27259 614	.13794 644
.5	872 582	13 22	.5	9622 586	1674 208	.5	.18484 598	6239 412	.5	.27567 616	.14116 654
2	1163 582	24 28	17	9915 586	1778 214	32	.18783 598	6445 420	47	.27875 616	.14443 662
.5	1454 582	38 34	.5	.10203 586	1885 220	.5	.19082 598	6655 428	.5	.28183 618	.14774 672
3	1745 582	55 40	18	.10501 586	1995 226	33	.19381 598	6869 434	48	.28492 618	.15110 680
.5	2036 582	75 46	.5	.10794 588	2103 232	.5	.19680 600	7086 440	.5	.28801 618	.15450 690
4	2327 582	98 52	19	.11088 588	2224 240	34	.19980 600	7306 448	49	.29110 620	.15795 700
.5	2618 582	124 58	.5	.11382 588	2344 248	.5	.20280 602	7530 456	.5	.29420 620	.16145 710
5	2909 584	153 64	20	.11676 590	2468 254	35	.20581 600	7758 464	50	.29730 620	.16500 718
.5	0.03201 582	0.00185 70	.5	0.11971 588	0.02595 260	.5	0.20881 602	0.07990 472	.5	0.30040 620	0.16859 728
6	3492 582	220 76	21	.12265 588	2725 266	36	.21182 602	8226 480	51	.30350 624	.17223 738
.5	3783 582	258 82	.5	.12559 588	2858 272	.5	.21483 604	8466 490	.5	.30662 624	.17592 746
7	4074 582	299 88	22	.12853 590	2994 278	37	.21785 604	8711 498	52	.30974 624	.17965 756
.5	4365 584	343 96	.5	.13148 590	3133 286	.5	.22087 604	8960 504	.5	.31286 626	.18343 766
8	4657 584	391 100	23	.13443 590	3276 294	38	.22389 604	9212 512	53	.31599 626	.18726 778
.5	4949 582	441 106	.5	.13738 590	3423 300	.5	.22691 604	9468 518	.5	.31912 626	.19115 786
9	5240 584	494 114	24	.14033 590	3573 306	39	.22993 604	9727 526	54	.32225 628	.19508 796
.5	5532 582	551 120	.5	.14328 592	3726 314	.5	.23295 606	9990 534	.5	.32539 628	.19906 806
10	5823 584	611 126	25	.14624 592	3883 322	40	.23598 606	.10257 544	55	.32853 530	.20309 816
.5	0.06115 584	0.00674 132	.5	0.14920 592	0.04044 328	.5	0.23901 608	0.10529 552	.5	0.33168 630	0.20717 826
11	6407 582	740 138	26	.15216 592	4208 334	41	.24205 608	.10805 560	56	.33483 632	.21130 836
.5	6698 584	809 146	.5	.15512 592	4375 340	.5	.24509 610	.11085 568	.5	.33799 632	.21548 848
12	6990 584	882 152	27	.15808 592	4545 348	42	.24814 608	.11369 576	57	.34115 632	.21972 858
.5	7282 584	958 156	.5	.16104 594	4719 354	.5	.25118 610	.11657 584	.5	.34431 634	.22401 870
13	7574 584	1036 162	28	.16401 594	4896 362	43	.25423 610	.11949 592	58	.34748 634	.22836 880
.5	7866 586	1117 168	.5	.16698 594	5077 370	.5	.25728 612	.12245 602	.5	.35065 636	.23276 890
14	8159 584	1204 176	29	.16995 594	5262 376	44	.26034 612	.12543 612	59	.35383 636	.23721 900
.5	8451 586	1289 182	.5	.17292 596	5450 384	.5	.26340 612	.12852 620	.5	.35701 638	.24171 910
15	8744 584	1380 190	30	.17590 596	5642 390	45	.26646 612	.13162 628	60	.36020	.24626

TABLE II. COÖRDINATES $y = lC$; $x = l - lE$.

$$\phi = In^2 = I\frac{l^2}{l_1{}^2}.$$

$\phi°$	C	E	$\phi°$	C	E
1	0.00582 581	0.00003	31	0.17661 545	0.02888
2	1163 582	12	32	.18206 542	3075
3	1745 581	27	33	.18748 540	3267
4	2326 581	49	34	.19288 538	3464
5	2907 581	76	35	.19826 535	3667
6	3488 580	110	36	.20361 533	3876
7	4068 580	149	37	.20894 530	4090
8	4648 579	195	38	.21424 526	4310
9	5227 578	246	39	.21950 524	4535
10	5805 578	304	40	.22474 521	4765
11	0.06383 576	0.00368	41	0.22995 518	0.05001
12	6959 576	438	42	.23513 515	5241
13	7535 575	514	43	.24028 512	5487
14	8110 574	595	44	.24540 509	5739
15	8684 573	683	45	.25049 505	5995
16	9257 571	777	46	.25554 503	6256
17	9828 570	877	47	.26057 499	6523
18	.10398 569	983	48	.26556 496	6794
19	.10967 568	1094	49	.27052 492	7070
20	.11535 566	1212	50	.27544 488	7352
21	0.12101 564	0.01335	51	0.28032 486	0.07638
22	.12665 563	1464	52	.28518 482	7929
23	.13228 561	1599	53	.29000 478	8224
24	.13789 559	1740	54	.29478 474	8525
25	.14348 557	1887	55	.29952 471	8830
26	.14905 556	2040	56	.30423 467	9140
27	.15461 553	2198	57	.30890 462	9454
28	.16014 551	2362	58	.31352 459	9773
29	.16565 549	2531	59	.31811 455	.10096
30	.17114 547	2707	60	.32266	.10423

TABLE III. DEFLECTION ANGLES $d = \frac{1}{3}A - B$.

n	ϕ for $I \div 3 = 1$.	A	Inst. at sta. $n'' = 0$. B For $I \div 3 =$								
			4°	6°	8°	10°	12°	14°	16°	18°	20°
0	.00	.00									
.05	.0075	.0025									
.1	.03	.01									
.15	.0675	.0225									
.2	.12	.04									
.25	.1875	.0625									
.3	.27	.09									
.35	.3675	.1225									
.4	.48	.16									1
.45	.6075	.2025						1	1	1	1
.5	.75	.25					1	1	1	2	3
.55	.9075	.3025				1	1	2	3	4	5
.6	1.08	.36			1	1	2	3	4	6	9
.65	1.2675	.4225			1	2	3	5	7	10	14
.7	1.47	.49		1	1	3	5	7	11	16	22
.75	1.6875	.5625		1	2	4	7	11	17	24	33
.8	1.92	.64		1	3	6	11	17	25	36	49
.85	2.1675	.7225	1	2	4	9	15	24	36	52	71
.9	2.43	.81	1	3	6	12	21	34	51	73	100
.95	2.7075	.9025	1	4	9	17	30	47	71	102	139
1.	3.	1.	1	5	12	23	41	64	97	138	191

TABLE III.—DEFLECTION ANGLES $d = \frac{I}{3}A - B$. (See also Table IV.)

n	ϕ for $I \div 3 = 1$.	Inst. at sta. $n'' = 1$.										Inst. at sta. $n'' = .5$.				Inst. at sta. $n'' = .25$.							Inst. at sta. $n'' = .75$.						n
		A	B For $I \div 3 =$									A	B For $I \div 3 =$			A	B For $I \div 3 =$						A	B For $I \div 3 =$					
			4°	6°	8°	10°	12°	14°	16°	18°	20°		8°	12°	16°	20°	4°	8°	12°	14°	16°	18°	20°		6°	10°	14°	16°	18°	20°	
.0	.00	1.	1	5	12	23	41	64	97	138	191	.25		1	1	3	.0625							.5625	1	4	11	17	24	33	0
.05	.0075	1.0525	1	4	11	20	36	58	86	124	171	.2775			1	2	.0775							.6025	1	4	10	15	21	29	.05
.1	.03	1.11	1	4	9	18	32	51	76	109	150	.31			1	2	.0975							.6475	1	3	8	12	17	24	.1
.15	.0675	1.1725	1	3	8	16	28	44	66	95	130	.3475			1	1	.1225							.6975	1	2	7	10	14	20	.15
.2	.12	1.24	1	3	7	13	23	37	56	80	110	.39				1	.1525							.7525		2	6	8	12	16	.2
.25	.1875	1.3125	1	2	6	11	20	31	47	68	93	.4375					.1875							.8125		1	4	6	9	12	.25
.3	.27	1.39	1	2	5	9	16	26	39	56	77	.49					.2275							.8775		1	3	4	6	9	.3
.35	.3675	1.4725		2	4	7	13	21	31	44	61	.5475					.2725							.9475		1	2	3	4	6	.35
.4	.48	1.56		1	3	6	10	16	24	34	47	.61					.3225							1.0225			1	2	3	4	.4
.45	.6075	1.6525		1	2	4	8	12	18	26	36	.6775					.3775							1.1025			1	1	2	2	.45
.5	.75	1.75		1	2	3	6	9	14	19	26	.75					.4375							1.1875				1	1	1	.5
.55	.9075	1.8525			1	2	4	6	9	13	19	.8275					.5025						1	1	1.2775						.55
.6	1.08	1.96			1	1	3	4	6	9	13	.91					.5725					1	1	1	1.3725						.6
.65	1.2675	2.0725				1	2	3	4	6	8	.9975					.6475			1	1	2	3	4	1.4725						.65
.7	1.47	2.19					1	1	2	3	5	1.09					.7275	1	2	3	4	6	8	1.5775						.7
.75	1.6875	2.3125						1	1	2	3	1.1875			1	1	.8125	1	3	4	6	9	12	1.6875						.75
.8	1.92	2.44								1	1	1.29		1	1	2	.9025	1	4	6	9	13	18	1.8025						.8
.85	2.1675	2.5725									1	1.3975		1	3	6	.9975	2	6	10	15	21	29	1.9225						.85
.9	2.43	2.71										1.51	1	2	5	9	1.0975	3	9	15	22	32	44	2.0475					1	.9
.95	2.7075	2.8525										1.6275	1	3	8	16	1.2025	1	4	14	22	33	46	65	2.1775				1	1	.95
1.	3.	3.										1.75	2	6	14	26	1.3125	1	6	20	31	47	68	93	2.3125			1	1	2	3	1.

TABLE IV.—DEFLECTIONS. Zero on first tangent.

100-FOOT CHORDS PER 1°.

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 1	C.P. 2	C.P. 3	C.P. 4	C.P. 5	C.P. 6	C.P. 7	C.P. 8	C.P. 9	C.P. 10	C.P. 11	I	Long Ch'rd
1°	.07	100	1	10′												0° 30′	100
2	.58	200	2	40	1° 10′											2 00	200
3	1.96	300	3	1° 30	2 10	3° 10′										4 30	299.9
4	4.65	400	4	2 40	3 30	4 40	6° 10′									8 00	399.6
5	9.07	500	5	4 09.9	5 09.9	6 30	8 10	10° 10′								12 30	499.0
6	15.64	600	6	5 59.7	7 09.8	8 39.9	10 29.9	12 40	15° 10′							18 00	597.4
7	24.78	700	7	8 09.2	9 29.4	11 09.7	13 09.8	15 29.9	18 10	21° 10′						24 30	694.3
8	36.81	800	8	10 38.3	12 08.7	13 59.2	16 09.6	19 39 8	21 29.9	24 40	28° 10′					32 00	789.0

80-FOOT CHORDS PER 1°.

1	.05	80	1	08												24	80
2	.37	160	2	32	56											1 36	160
3	1.26	240	3	1 12	1 44	2 32										3 36	240
4	2.98	320	4	2 08	2 48	3 44	4 56									6 24	319.8
5	5.81	400	5	3 20	4 08	5 12	6 32	8 08								10 00	399.5
6	10.03	480	6	4 47.8	5 43.9	6 55.9	8 24	10 08	12 08							14 24	478.7
7	15.89	560	7	6 31.6	7 35.6	8 55.8	10 31.9	12 24	14 32	16 56						19 36	557.1
8	23.66	640	8	8 30.1	9 43.3	11 11.6	12 55.7	14 55.9	17 12	19 44	22 32					25 36	634.3
9	33.53	720	9	10 46.2	12 06.6	13 43 0	15 35.4	17 43.6	20 07.9	22 48	25 44	28° 56′				32 24	709.8
10	45.73	800	10	13 16.6	14 46.0	16 30.5	18 30.9	20 47.3	23 19.6	26 07.8	29 11.9	32 32	36° 08′			40 00	782.6

60-FOOT CHORDS PER 1°.

1	.03	60	1	06												18	60
2	.21	120	2	24	42											1 12	120
3	.71	180	3	54	1 18	1 54										2 42	180
4	1.67	240	4	1 36	2 06	2 48	3 42									4 48	239.9
5	3.27	300	5	2 30	3 06	3 54	4 54	6 06								7 30	299.8
6	5.64	360	6	3 36	4 18	5 12	6 18	7 36	9 06							10 48	359.4
7	8.95	420	7	4 51.8	5 41.9	6 41.9	7 54	9 18	10 54	12 42						14 42	418.7
8	13.34	480	8	6 23.6	7 17.7	8 23.8	9 41.9	11 11.9	12 54	14 48	16 54					19 12	477.6
9	18.95	540	9	8 05.3	9 05.5	10 17.6	11 41.7	13 17.9	15 05.9	17 06	19 18	21 42				24 18	535.7
10	25.91	600	10	9 58.6	11 04.9	12 23.2	13 53.5	15 35.6	17 29.8	19 35.9	21 54	24 24	27 06			30 00	592.8
11	34.34	660	11	12 03.5	13 16.0	14 40.5	16 16.9	18 05.3	20 05.5	22 17.8	24 41.9	27 18	30 06	33° 06′		36 18	648.4
12	44.31	720	12	14 19.7	15 38.5	17 09.2	18 52.0	20 46.6	22 53.0	25 11.4	27 41.7	30 23.9	33 17.9	36 24	39° 42′	43 12	701.9

TABLE IV.—DEFLECTIONS. Zero on first tangent.

50-FOOT CHORDS PER 1°.

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 1	C.P. 2	C.P. 3	C.P. 4	C.P. 5	C.P. 6	C.P. 7	C.P. 8	C.P. 9	C.P. 10	C.P. 11	I	Long Ch'rd
2°	.14	100	2	20′	35′											1° 00′	100
3	.37	150	3	45	1° 05	1° 35′										2 15	150
4	1.16	200	4	1° 20	1 45	2 20	3° 05′									4 00	200
5	2.27	250	5	2 05	2 35	3 15	4 05	5° 05′								6 15	249.9
6	3.92	300	6	3 00	3 35	4 20	5 15	6 20	7° 35′							9 00	299.7
7	6.58	350	7	4 04.9	4 44.9	5 34.9	6 35	7 45	9 05	10° 35′						12 15	349.3
8	9.27	400	8	5 19.8	6 04.9	6 59.9	8 05	9 20	10 45	12 20	14° 05′					16 00	398.6
9	13.19	450	9	6 44.5	7 34.6	8 34.7	9 44.8	11 04.9	12 34.9	14 15	16 05	18° 05′				20 15	447.4
10	18 05	500	10	8 19.2	9 14.4	10 19.5	11 34.6	12 59.8	14 34.9	16 19.9	18 15	20 20	22° 35′			25 00	495.8
11	23 96	550	11	10 03.6	11 03.9	12 14.2	13 34.3	15 04.6	16 44.8	18 34.9	20 34.9	22 45	25 05	27° 35′		30 15	543.3
12	30.96	600	12	11 57.5	13 03.0	14 18.5	15 43.8	17 19.1	19 04.4	20 59.6	23 04.8	25 19.9	27 45	30 20	33° 05′	36 00	589.6

40-FOOT CHORDS PER 1°.

					C.P. 2	C.P. 4	C.P. 5	C.P. 6	C.P. 7	C.P. 8	C.P. 9	C.P. 10	C.P. 11	C.P. 12	C.P. 13		
2	.09	80	2	16												48	80
3	.32	120	3	36	1 16											1 48	120
4	.75	160	4	1 04	1 52											3 12	160
5	1.45	200	5	1 40	2 36	4 04										5 00	199.9
6	2.51	240	6	2 24	3 28	5 04	6 04									7 12	239.8
7	3.99	280	7	3 16	4 28	6 12	7 16	8 28								9 48	279.7
8	5.95	320	8	4 15.9	5 36	7 28	8 36	9 52	11 16							12 48	319.3
9	8.45	360	9	5 23.8	6 51.9	8 52	10 04	11 24	12 52	14 28						16 12	358.7
10	11.57	400	10	6 39.6	8 15.8	10 23.9	11 39.9	13 04	14 36	16 16	18 04					20 00	397.9
11	15.37	440	11	8 03.3	9 47.5	12 03.8	13 23.9	14 51.9	16 28	18 12	20 04	22 04				24 12	436.5
12	19.92	480	12	9 34.7	11 27.3	13 51.6	15 15.7	16 47.8	18 27.8	20 16	21 12	24 16	26 28			28 48	474.6
13	25.23	520	13	11 13.9	13 14.7	15 47.3	17 15.6	18 51.7	20 35.8	22 27.9	24 28	26 36	28 52	31 16		33 48	512.0
14	31.39	560	14	13 00.8	15 09.8	17 50.7	19 23.0	21 03.3	22 51.6	24 47.6	26 51.8	29 03.9	31 24	33 52	36 28	39 12	548.5

35-FOOT CHORS PER 1°.

2	.07	70	2	14												42	70
3	.24	105	3	31.5	1 06.5											1 34.5	105
4	.57	140	4	56	1 38											2 48	140
5	1.12	175	5	1 27.5	2 16.5	3 33.5										4 22.5	175
6	1.93	210	6	2 06	3 02	4 26	5 18.5									6 18	210
7	3.06	245	7	2 51.5	3 54.5	5 25.5	6 21.5	7 24.5								8 34.5	244.8
8	4 56	280	8	3 44	4 54	6 32	7 31.5	8 38	9 51.5							11 12	279.5

TABLE IV.—DEFLECTIONS. Zero on first tangent.

35-FOOT CHORDS PER 1°.—*Continued.*

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 2	C.P. 4	C.P. 5	C.P. 6	C.P. 7	C.P. 8	C.P. 9	C.P. 10	C.P. 11	C.P. 12	C.P. 13	I	Long Ch'rd
9°	6.48	315	9	4° 43′.4	6° 00′.4	7° 45′.5	8° 48′.5	9° 58′.5	11° 15′.5	12° 39′.5						14° 10′.5	314.2
10	8.86	350	10	5 49.7	7 13.8	9 05.9	10 12.4	11 26	12 46.5	14 14	15° 48′.5					17 30	348.6
11	11.77	385	11	7 03.0	8 34.2	10 33.3	11 43.4	13 00.4	14 24.5	15 55.5	17 33.5	19° 18 .5				21 10.5	382.7
12	15.26	420	12	8 23.2	10 01.5	12 07.7	13 21.3	14 41.9	16 09.4	17 44	19 25.5	21 14	23° 09′.5			25 12	416.3
13	19.37	455	13	9 50.2	11 35.6	13 49.0	15 06.1	16 30.3	18 01.4	19 39.4	21 24.5	23 16.5	25 15.5	27° 21′.5		29 34.5	449.6
14	24.12	490	14	11 23.8	13 16.6	15 37.1	16 57.9	18 25.6	20 00.2	21 41.9	23 30.4	25 26.5	27 28.5	29 38	31° 54′.5	34 18	482.2

30-FOOT CHORDS PER 1°.

								C.P. 6	C.P. 8	C.P. 10	C.P. 12	C.P. 13	C.P. 14	C.P. 15	C.P. 16	C.P. 17			
2	.05	60	2	12													36	60
3	.18	90	3	27	57												1 21	90
4	.42	120	4	48	1 24												2 24	120
5	.82	150	5	1 15	1 57	3 03											3 45	150
6	1.42	180	6	1 48	2 36	3 48											5 24	179.9
7	2.25	210	7	2 27	3 21	4 39	6 21										7 21	209.9
8	3.35	240	8	3 12	4 12	5 36	7 24										9 36	239.7
9	4.76	270	9	4 02.9	5 09	6 39	8 33	10 51									12 09	269.5
10	6.52	300	10	4 59.8	6 11.9	7 48	9 48	12 12									15 00	299.0
11	8 67	330	11	6 02.7	7 20.8	9 02.9	11 09	13 39	16 33								18 09	328.5
12	11.24	360	12	7 11.5	8 35.7	10 23.8	12 35.9	15 12	18 12								21 36	357.7
13	14.27	390	13	8 26.1	9 56.4	11 50.6	14 08.9	16 50.9	19 57	23 27							25 21	386.6
14	17.78	420	14	9 46.7	11 23.1	13 23.5	15 47.8	18 35.9	21 48	25 24	27 21						29 24	415.1
15	21.80	450	15	11 13.0	12 55.6	15 02.1	17 32.5	20 26.8	23 44.9	27 27	29 27	31 33					33 45	443.1
16	26.36	480	16	12 45.0	14 33.8	16 46.5	19 23.2	22 23.6	25 47.8	29 36	31 39	33 48	36 03				38 24	470.5
17	31.48	510	17	14 22.7	16 17.8	18 36.8	21 19.6	24 26.3	27 56.7	31 50.9	33 57	36 09	38 27	40 51			43 21	491.1
18	37.19	540	18	16 05.9	18 07.3	20 32.7	23 21.8	26 34.8	30 11.4	34 11.8	36 20.9	38 36	40 57	43 24	45 57		48 36	522.9

25-FOOT CHORDS PER 1°.

2	.03	50	2	10													30	50
3	.13	75	3	22.5	47.5												1 07.5	75
4	.29	100	4	40	1 10												2 00	100
5	.56	125	5	1 02.5	1 37.5	2 32.5											3 07.5	125
6	.98	150	6	1 30	2 10	3 10											4 30	149.9
7	1.56	175	7	2 02.5	2 47.5	3 52.5	5 17.5										6 07.5	174.9
8	2.32	200	8	2 40	3 30	4 40	6 10										8 00	199.8
9	3.30	225	9	3 22.4	4 17.4	5 32.5	7 07.5	9 02.5									10 07.5	224.7
10	4.53	250	10	4 09.9	5 09.9	6 30	8 10	10 10									12 30	249.5

TABLE IV.—DEFLECTIONS. Zero on first tangent.

25-FOOT CHORDS PER 1°.—*Continued.*

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 2	C.P. 4	C.P. 6	C.P. 8	C.P. 10	C.P. 12	C.P. 14	C.P. 16	C.P. 17	C.P. 18	C.P. 19	I	Long Ch'rd
11°	6.03	275	11	5° 02′ 3	6° 07.′4	7° 32′.4	9° 17′.5	11° 22′.5	13° 47′.5							15° 07′.5	274.2
12	7.82	300	12	5 59.7	7 09.8	8 39.9	10 29.9	12 40	15 10							18 00	298.7
13	9.93	325	13	7 02 0	8 17.2	9 52.3	11 47.4	14 02.5	16 37.5	19° 32′.5						21 07.5	223.0
14	12.39	350	14	8 09.2	9 29.4	11 09.7	13 09.8	15 29 9	18 10	21 10						24 30	347.1
15	15.19	375	15	9 21.3	10 46.6	12 32 0	14 37.2	17 02.4	19 47.4	22 52.5	26° 17′.5					28 07.5	370.9
16	18.40	400	16	10 38 3	12 08 7	13 59.2	16 09.6	18 39.8	21 29.9	24 40	28 10					32 00	394.5
17	22.02	425	17	12 00.0	13 35.5	15 31.2	17 46.8	20 22.1	23 17.3	26 32.4	30 07.5	34° 02′.5				36 07.5	417.4
18	26.03	450	18	13 26.5	15 07.3	17 08.1	19 28.7	22 09.3	25 09 7	28 29.9	32 10	36 10	38° 17′.5			40 30	440 1
19	30.48	475	19	14 57.6	16 43.7	18 49.7	21 15.6	24 01.4	27 07.0	30 32 2	34 17.4	38 22.5	40 32.5	42° 47′ 5		45 07.5	463.1
20	35 37	500	20	16 33.3	18 24.8	20 36.2	23 07.3	25 58.4	29 09.0	32 39.6	36 29.9	40 40	42 52.5	45 10	47° 32′.5	50 00	483.2

20-FOOT CHORDS PER 1°.

																C.P. 18	C.P. 20	C.P. 22		
2	.02	40	2	08															24	40
3	.08	60	3	18	38														54	60
4	.18	80	4	32	56														1 36	80
5	.36	100	5	50	1 18	2 02													2 30	100
6	.63	120	6	1 12	1 44	2 32													3 36	120
7	1.00	140	7	1 38	2 14	3 06	4 14												4 54	140
8	1.49	160	8	2 08	2 48	3 44	4 56												6 24	159.9
9	2.12	180	9	2 42	3 26	4 26	5 42	7 14											8 06	179.8
10	2.91	200	10	3 20	4 08	5 12	6 32	8 08											10 00	199.7
11	3.86	220	11	4 01.9	4 53.9	6 02	7 26	9 06	11 02										12 06	219.5
12	5.01	240	12	4 47.8	5 43.9	6 55.9	8 24	10 08	12 08										14 24	239.3
13	6.36	260	13	5 37.8	6 37.8	7 53.9	9 25.9	11 14	13 18	15 38									16 54	258.9
14	7.95	280	14	6 31.6	7 35.6	8 55.8	10 31.9	12 24	14 32	16 56									19 36	278.5
15	9.76	300	15	7 29.4	8 37.6	10 01.7	11 41.8	13 37.9	15 50	18 18	21 02								22 30	297.9
16	11.82	320	16	8 30.1	9 43.3	11 11.6	12 55.7	14 55 9	17 12	19 44	22 32								25 36	317.2
17	14.16	340	17	9 36.7	10 53.1	12 25.3	14 13.6	16 17.8	18 37.9	21 14	24 06	27 14							28 54	336.2
18	16.76	360	18	10 46.2	12 06.6	13 43.0	15 35.4	17 43.6	20 07.9	22 48	25 44	28 56							32 24	355.0
19	19.66	380	19	11 59.6	13 24.1	15 04.6	17 01.0	19 13.4	21 41.7	24 25.8	27 25.9	30 42	34 14						36 06	373.3
20	22.86	400	20	13 16.6	14 46.0	16 30.5	18 30.9	20 47.3	23 19.6	26 07.8	29 11.9	32 32	36 08						40 00	391.4
21	26.37	420	21	14 37.5	16 10.4	17 59.8	20 04.1	22 24.7	25 01.2	27 53.7	31 01.9	34 26	38 06	42 02					44 06	409.1
22	30.18	440	22	16 02.1	17 39.5	19 32.6	21 41.4	24 06.4	26 47.0	29 43.4	32 55.7	36 23.9	40 08	44 08					48 24	426.2
23	34.32	460	23	17 30.1	19 11.6	21 09.0	23 22.3	25 51.5	28 36.4	31 37.0	34 53.5	38 25.8	42 13.9	46 18	50 38				52 54	442.8
24	38.77	480	24	19 01.7	20 47.6	22 49.4	25 07.0	27 40.4	30 29.6	33 34.6	36 55 2	40 31.6	44 23.8	48 31.9	52 56				57 36	458.8

TABLE IV.—DEFLECTIONS. Zero on first tangent.

30-FOOT CHORDS PER 2°.

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 1	C.P. 2	C.P. 3	C.P. 4	C.P. 5	C.P. 6	C.P. 7	C.P. 8	C.P. 9	C.P. 10	C.P. 11	I	Long Ch'rd
2°	.01	30	1	06′												18′	30
3	.04	45	1.5	13.5	28′.5											40.5	45
4	.10	60	2	24	42											1° 12	60
5	.21	75	2.5	37.5	58.5	1° 31′.5										1 52.5	75
6	.36	90	3	54	1° 18	1 54										2 42	90
7	.56	105	3.5	1° 13.5	1 40.5	2 19.5	3° 10′.5									3 40.5	105
8	.84	120	4	1 36	2 06	2 48	3 42									4 48	119.9
9	1.19	135	4.5	2 01.5	2 34.5	3 19.5	4 16.5	5° 25′.5								6 04.5	134.9
10	1.64	150	5	2 30	3 06	3 54	4 54	6 06								7 30	149.9
11	2.18	165	5.5	3 01.5	3 40.5	4 31.5	5 34.5	6 49.5	8° 16′.5							9 04.5	164.8
12	2.82	180	6	3 36	4 18	5 12	6 18	7 36	9 06							10 48	179.7
13	3.59	195	6.5	4 13.4	4 58.4	5 55.4	7 04.5	8 25.5	9 58.5	11° 43′.5						12 40.5	194 6
14	4.48	210	7	4 51.8	5 41.9	6 41.9	7 54	9 18	10 54	12 42						14 42	209.4
15	5.51	225	7.5	5 37.3	6 28.3	7 31.4	8 46 4	10 13.5	11 52.5	13 43.5	15° 46′.5					16 52.5	224.1
16	6.68	240	8	6 23.6	7 17.7	8 23.8	9 41 9	11 11.9	12 54	14 48	16 54					19 12	238.8
17	8.00	255	8.5	7 12.9	8 10.1	9 19.2	10 40.3	12 13.4	13 58.5	15 55.5	18 04.5	20° 25′.5				21 40.5	253.3
18	9.47	270	9	8 05.3	9 05.5	10 17.6	11 41.7	13 17.9	15 05.9	17 06	19 18	21 42				24 18	267.8
19	11.12	285	9.5	9 00.4	10 03.7	11 18.9	12 46.1	14 25.3	16 16.4	18 19.5	20 34.5	23 01.5	25° 40′.5			27 04.5	282.1
20	12.96	300	10	9 58.6	11 04.9	12 23.2	13 53.5	15 35.6	17 29.8	19 35.9	21 54	24 24	27 06			30 00	296.4
21	14.97	315	10.5	10 59.6	12 09.0	13 30.4	15 03.7	16 49.0	18 46.2	20 55.4	23 16.5	25 49.5	28 34.5	31° 31′.5		33 04.5	310.4
22	17.17	330	11	12 03.5	13 16.0	14 40.5	16 16.9	18 05.3	20 05.5	22 17.8	24 41.9	27 18	30 06	33 06		36 18	324.2
23	19.56	345	11.5	13 10.2	14 25.7	15 53.8	17 33.0	19 24.3	21 27.8	23 43.1	26 10.3	28 49.4	31 40.5	34 43.5	37° 58′.5	39 40.5	337.7
24	22.15	360	12	14 19.7	15 38.5	17 09.2	18 52.0	20 46.6	22 53.0	25 11.4	27 41.7	30 23.9	33 17.9	36 24	39 42	43 12	351.0

20-FOOT CHORDS PER 2°.

2	.01	20	1	04												12	20
3	.02	30	1.5	09	19											27	30
4	.04	40	2	16	28											48	40
5	.09	50	2.5	25	39	1 01										1 15	50
6	.16	60	3	36	52	1 16										1 48	60
7	.25	70	3.5	49	1 07	1 33	2 07									2 27	70
8	.37	80	4	1 04	1 24	1 52	2 28									3 12	80
9	.54	90	4.5	1 21	1 43	2 13	2 51	3 37								4 03	90
10	.73	100	5	1 40	2 04	2 36	3 16	4 04								5 00	100

TABLE IV.—DEFLECTIONS. Zero on first tangent.

20-FOOT CHORDS PER 2°.—*Continued.*

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 2	C.P. 4	C.P. 6	C.P. 8	C.P. 9	C.P. 10	C.P. 11	C.P. 12	C.P. 13	C.P. 14	C.P. 15	l	Long Ch'rd
12°	1.26	120	6	2° 24′	3° 28′	5° 04′										7° 12′	119.9
14	1.99	140	7	3 16	4 28	6 12	8° 28′									9 48	139.8
16	2.97	160	8	4 15.9	5 36	7 28	9 52									12 48	159.6
18	4.23	180	9	5 23.8	6 51.9	8 52	11 24	14° 28′								16 12	179.4
20	5.79	200	10	6 39.6	8 15.8	10 23.9	13 04	16 16	18° 04′							20 00	198.9
22	7.69	220	11	8 03.3	9 47.5	12 03.8	14 51.9	18 12	20 04	22° 04′						24 12	218.3
24	9.97	240	12	9 34.7	11 27.3	13 51.6	16 47.8	20 16	22 12	24 16	26° 28′					28 48	237.3
26	12.61	260	13	11 13.9	13 14.7	15 47.3	18 51.7	22 27.9	24 28	26 36	28 52	31° 16′				33 48	256.0
28	15.69	280	14	13 00.8	15 09.8	17 50.7	21 03.3	24 47.6	26 51.8	29 03.9	31 24	33 52	36° 28′			39 12	274.2
30	19.20	300	15	14 55.2	17 12.5	20 01.7	23 22.8	27 15 5	29 23.7	31 39.8	34 03.9	36 36	39 16	42° 04′		45 00	291.8
32	23.15	320	16	16 57.0	19 22.7	22 20.5	25 50.0	29 51.0	32 03.4	34 23.6	36 51.8	39 27.9	42 12	45 04	48° 04′	51 12	308.8

10-FOOT CHORDS PER 2°.

					C.P. 4	C.P. 6	C.P. 8	C.P. 10	C.P. 12	C.P. 14	C.P. 16	C.P. 18	C.P. 19	C.P. 20	C.P. 21		
4	.01	20	2	08												24	20
6	.04	30	3	18												54	30
8	.09	40	4	32												1 36	40
10	.18	50	5	50	2 02											2 30	50
12	.32	60	6	1 12	2 32											3 36	60
14	.50	70	7	1 38	3 06	4 14										4 54	70
16	.75	80	8	2 08	3 44	4 56										6 24	80
18	1.06	90	9	2 42	4 26	5 42	7 14									8 06	89.9
20	1.45	100	10	3 20	5 12	6 32	8 08									10 00	99.9
22	1.93	110	11	4 01.9	6 02	7 26	9 06	11 02								12 06	109.8
24	2.51	120	12	4 47.8	6 55.9	8 24	10 08	12 08								14 24	119.6
26	3.18	130	13	5 37.8	7 53.9	9 26	11 14	13 18	15 38							16 54	129.5
28	3.97	140	14	6 31.6	8 55.8	10 31.9	12 24	14 32	16 56							19 36	139.3
30	4.88	150	15	7 29.4	10 01.7	11 41.9	13 37.9	15 50	18 18	21 02						22 30	149.0
32	5.91	160	16	8 30.1	11 11.6	12 55.7	14 55.9	17 12	19 44	22 32						25 36	158 5
34	7.08	170	17	9 36.7	12 25.3	14 13.6	16 17.8	18 37.9	21 14	24 06	27 14					28 54	168.1
36	8.38	180	18	10 46.2	13 43.0	15 35.4	17 43.6	20 07.9	22 48	25 44	28 56					32 24	177.4
38	9.83	190	19	11 59.6	15 04.6	17 01.0	19 13.4	21 41.7	24 25.8	27 25.9	30 42	34 14				36 06	186.7
40	11.43	200	20	13 16.6	16 30.5	18 30.9	20 47.3	23 19.6	26 07.8	29 11.9	32 32	36 08	38 02			40 00	195.6
42	13.18	210	21	14 37.5	17 59.3	20 04.1	22 24.7	25 01.2	27 53.7	31 01.9	34 26	38 06	40 02	42 02		44 06	204 5
44	15.09	220	22	16 02.1	19 32.6	21 41.4	24 06.4	26 47.0	29 43.4	32 55.7	36 23.9	40 08	42 06	44 08	46 14	48 24	213.1

TABLE IV.—DEFLECTIONS. Zero on first tangent.

10-FOOT CHORDS PER 4°.

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 2	C.P. 4	C.P. 6	C.P. 8	C.P. 10	C.P. 11	C.P. 12	C.P. 13	C.P. 14	C.P. 15	C.P. 16	I	Long Ch'rd
8°	.02	20	2	16′												48′	20
12	.08	30	3	36	1° 16′											1° 48	30
16	.19	40	4	1° 04	1 52											3 12	40
20	.36	50	5	1 40	2 36	4° 04′										5 00	50
24	.63	60	6	2 24	3 28	5 01										7 12	60
28	1.00	70	7	3 16	4 28	6 12	8° 28′									9 48	69.9
32	1.49	80	8	4 15.9	5 36	7 28	9 52									12 48	79.8
36	2.11	90	9	5 23.8	6 51.9	8 52	11 24	14° 28′								16 12	89.7
40	2.89	100	10	6 39.6	8 15.8	10 23.9	13 04	16 16								20 00	99.5
44	3.84	110	11	8 03.3	9 47.5	12 03.8	14 51.9	18 12	22° 04′							24 12	109 1
48	4.98	120	12	9 34.7	11 27.3	13 51.6	16 47.8	20 16	24 16	26° 28′						28 48	118.7
52	6.31	130	13	11 13.9	13 14.7	15 47.3	18 51.7	22 27.9	26 36	28 52	31° 16′					33 48	128.0
56	7.85	140	14	13 00.8	15 09.8	17 50.7	21 03.3	24 47.6	29 03.9	31 24	33 52	36° 28′				39 12	137.1
60	9.60	150	15	14 55.2	17 12.5	20 01.7	23 22.8	27 15.5	31 39.8	34 03.9	36 36	39 16	42° 04′			45 00	145.9
64	11.58	160	16	16 57.0	19 22.7	22 20 5	25 50.0	29 51.0	34 23.6	36 51.8	39 27.9	42 12	45 04	48° 04′		51 12	154.5
68	13.78	170	17	19 05.7	21 40.3	24 46.7	28 24.8	32 34.3	37 15.2	39 47.6	42 27.8	45 15.9	48 11.9	51 16	54° 28′	57 48	162.4

5-FOOT CHORDS PER 4°.

					C.P. 4	C.P. 6	C.P. 8	C.P. 10	C.P. 12	C.P. 14	C.P. 16	C.P. 18	C.P. 20	C.P. 21	C.P. 22		
8	.01	10	2	08												24	10
16	.05	20	4	32												1 36	20
24	.16	30	6	1 12	2 32											3 36	30
32	.37	40	8	2 08	3 44	4 56										6 54	40
40	.73	50	10	3 20	5 12	6 32	8 08									10 00	49.9
48	1.25	60	12	4 47.8	6 55.9	8 24	10 08	12 08								14 24	59.8
56	1.99	70	14	6 31.6	8 55.8	10 31.9	12 24	14 32	16 56							19 36	69.6
60	2.44	75	15	7 29.4	10 01.7	11 41.9	13 37.9	15 50	18 18	21 02						22 30	74.5
64	2.96	80	16	8 30.1	11 11.6	12 55.7	14 55.9	17 12	19 44	22 32						25 36	79.3
68	3 54	85	17	9 36.7	12 25.3	14 13.6	16 17.8	18 37.9	21 14	24 06	27 14					28 54	84.0
72	4.19	90	18	10 46.2	13 43.0	15 35.4	17 43.6	20 07 9	22 48	25 44	28 56					32 24	88.7
76	4.91	95	19	11 59.6	15 04.6	17 01.0	19 13.4	21 41.7	24 25.8	27 25.9	30 42	34 14				36 06	93.3
80	5.71	100	20	13 16.6	16 30.5	18 30.9	20 47.3	23 19 6	26 07.8	29 11.9	32 32	36 08				40 00	97.8
84	6 59	105	21	14 37.5	17 59.3	20 04.1	22 24.7	25 01.2	27 53.7	31 01.9	34 26	38 06	42 02			44 06	102.3
88	7.54	110	22	16 03.1	19 32.6	21 41.4	24 06.4	26 47.0	29 43.4	32 55.7	36 23.9	40 08	44 08			48 24	106.6
92	8.58	115	23	17 30.1	21 09.0	23 22.3	25 57.5	28 36.4	31 37.0	34 53.5	38 25.8	42 13.9	46 18	48 26	50 38	52 54	110.7

TABLE IV.—DEFLECTIONS. Zero on first tangent.

5-FOOT CHORDS PER 8°.

D	F	l_1	Ch. Pt.	Instrument at Sta. 0	C.P. 2	C.P. 4	C.P. 6	C.P. 8	C.P. 10	C.P. 11	C.P. 12	C.P. 13	C.P. 14	C.P. 15	C.P. 16	I	Long Ch'rd
16°	.01	10	2	16′												48′	10
24	.04	15	3	36	1° 16′											1° 48	15
32	.09	20	4	1° 04	1 52											3 12	20
40	.18	25	5	1 40	2 36	4° 04′										5 00	25
48	.32	30	6	2 24	3 28	5 04										7 12	30
56	.50	35	7	3 16	4 28	6 12	8° 28′									9 48	35
64	.74	40	8	4 15.9	5 36	7 28	9 52									12 48	39.9
72	1.06	45	9	5 23.8	6 51.9	8 52	11 24	14° 28′								16 12	44.8
80	1.45	50	10	6 39.6	8 15.8	10 23.9	13 04	16 16								20 00	49.7
88	1.92	55	11	8 03.3	9 47.5	12 03.8	14 51.9	18 12	22° 04′							24 12	54.6
96	2.49	60	12	9 34 7	11 27.3	13 51.6	16 47.8	20 16	24 16	26° 28′						28 48	59.3
104	3.15	65	13	11 13.9	13 14.7	15 47.3	18 51.7	22 27.9	26 36	28 52	31° 16′					33 48	64.0
112	3.92	70	14	13 00.8	15 09.8	17 50.7	21 03.3	24 47.6	29 03.9	31 24	33 52	36° 28′				39 12	68.6
120	4.80	75	15	14 55.2	17 12.5	20 01.7	23 22.8	27 15.5	31 39.8	34 03.9	36 36	39 16	42° 04			45 00	73.0
128	5.79	80	16	16 57.0	19 22.7	22 20.5	25 50.0	29 51.0	34 23.6	36 51.8	39 27.9	42 12	45 04	48° 04′		51 12	77.2
136	6.89	85	17	19 05.7	21 40.3	24 46.7	28 24.8	32 34.3	37 15 2	39 47.6	42 27 8	45 15.9	48 11.9	51 16	54° 28′	57 48	81.2

5-FOOT CHORDS PER 16°.

					C.P. 1	C.P. 2	C.P. 3	C.P. 4	C.P. 5	C.P. 6	C.P. 7	C.P. 8					
16	.00	5	1	8												24	5
24	.01	7.5	1.5	18	38											54	7.5
32	.02	10	2	32	56											1 36	10
40	.04	12.5	2.5	50	1 18	2 02										2 30	12.5
48	.08	15	3	1 12	1 44	2 32										3 36	15
56	.12	17.5	3.5	1 38	2 14	3 06	4 14									4 54	17.5
64	.19	20	4	2 08	2 48	3 44	4 56									6 24	20
72	.26	22.5	4.5	2 42	3 26	4 26	5 42	7 14								8 06	22.5
80	.36	25	5	3 20	4 08	5 12	6 32	8 08								10 00	25
88	.48	27.5	5.5	4 01.9	4 53.9	6 02	7 26	9 06	11 02							12 06	27.5
96	.63	30	6	4 47.8	5 43.9	6 55.9	8 24	10 08	12 08							14 24	29.9
104	.80	32.5	6.5	5 37.8	6 37.8	7 53.9	9 25.9	11 14	13 18	15 38						16 54	32.4
112	.99	35	7	6 31.6	7 35.6	8 55.8	10 31.9	12 24	14 32	16 56						19 36	34.8
120	1.22	37.5	7.5	7 29.4	8 37.6	10 01.7	11 41.8	13 37.9	15 50	18 18	21 02					22 30	37.3
128	1.48	40	8	8 30.1	9 43.3	11 11.6	12 55.7	14 55.9	17 12	19 44	22 32					25 36	39.6
136	1.77	42.5	8.5	9 36.7	10 53.1	12 25.3	14 13.6	16 17.8	18 37.9	21 14	24 06	27 14				28 54	42.0

TABLE V.—RADII, AND DEFLECTION ANGLES IN MINUTES PER FOOT OF LENGTH.

Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.
0° 0′			0° 30′	11460.0	.15	1° 0′	5730.00	.3	1° 30′	3820.00	.45	2° 0′	2865.00	.6
1	343800.0	.005	31	11090.3	.155	1	5636.05	.305	31	3778.01	.455	1	2841.32	.605
2	171900.0	.01	32	10743.8	.16	2	5545.16	.31	32	3736.95	.46	2	2818.02	.61
3	114600.0	.015	33	10418.2	.165	3	5457.15	.315	33	3696.78	.465	3	2795.12	.615
4	85950.0	.02	34	10111.8	.17	4	5371.86	.32	34	3657.44	.47	4	2772.58	.62
5	68760.0	.025	35	9822.86	.175	5	5289.23	.325	35	3618.96	.475	5	2750.40	.625
6	57300.0	.03	36	9550.00	.18	6	5209.09	.33	36	3581.25	.48	6	2728.57	.63
7	49114.3	.035	37	9291.89	.185	7	5131.35	.335	37	3544.34	.485	7	2707.09	.635
8	42975.0	.04	38	9047.38	.19	8	5055.88	.34	38	3508.17	.49	8	2685 93	.64
9	38200.0	.045	39	8815.40	.195	9	4982.61	.345	39	3472.73	.495	9	2665.12	.645
10	34380.0	.05	40	8595.00	.2	10	4911.43	.35	40	3438.00	.5	10	2644.62	.65
11	31254 5	.055	41	8385.37	.205	11	4842.26	.355	41	3403 96	.505	11	2624.43	.655
12	28650.0	.06	42	8185.71	.21	12	4775.00	.36	42	3370.59	.51	12	2604.55	.66
13	26446.2	.065	43	7995.35	.215	13	4709.58	.365	43	3337.86	.515	13	2584.97	.665
14	24557.1	.07	44	7813 65	.22	14	4645.94	.37	44	3305.77	.52	14	2565.68	.67
15	22920.0	.075	45	7640.00	.225	15	4584.00	.375	45	3274.29	.525	15	2546.67	.675
16	21487.5	.08	46	7473.91	.23	16	4523.69	.38	46	3243.40	.53	16	2527.94	.68
17	20223.5	.085	47	7314.89	.235	17	4464.95	.385	47	3213.09	.535	17	2509.50	.685
18	19100.0	.09	48	7162.50	.24	18	4407.70	.39	48	3183 33	.54	18	2491.31	.69
19	18094.7	.095	49	7016.33	.245	19	4351.90	.395	49	3154.13	.545	19	2473.39	.695
20	17190.0	.10	50	6876.00	.25	20	4297.50	.4	50	3125.45	.55	20	2455.72	.7
21	16371.4	.105	51	6741.18	255	21	4244.45	.405	51	3097.30	.555	21	2438.30	.705
22	15627.2	.11	52	6611.54	.26	22	4192.68	.41	52	3069.64	.56	22	2421.13	.71
23	14947.8	.115	53	6486.79	.265	23	4142.18	.415	53	3042.48	.565	23	2404.20	.715
24	14325.0	.12	54	6366.67	.27	24	4092.86	.42	54	3015.79	.57	24	2387.50	.72
25	13752.0	.125	55	6250.90	.275	25	4044.70	.425	55	2989.56	.575	25	2371 04	.725
26	13223.1	.13	56	6139.29	.28	26	3997.68	.43	56	2963.79	.58	26	2354.79	.73
27	12733.3	.135	57	6031.58	.285	27	3951.73	.435	57	2938.46	.585	27	2338.78	.735
28	12278.6	.14	58	5927.59	.29	28	3906.82	.44	58	2913.56	.59	28	2322.97	.74
29	11855.1	.145	59	5827.12	.295	29	3862.92	.445	59	2889.08	.595	29	2307.38	.745
30	11460.0	.15	60	5730.00	.3	30	3820.00	.45	60	2865.00	.6	30	2292.00	.75

TABLE V.—RADII, AND DEFLECTION ANGLES IN MINUTES PER FOOT OF LENGTH.

Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.
2° 30′	2292.00	.75	3° 0′	1910 00	.9	3° 30′	1637.15	1 05	4° 0′	1432.50	1.2	4° 30′	1273.33	1.35
31	2276.82	.755	1	1899 45	.905	31	1629.38	1.055	1	1426.55	1.205	31	1268.63	1.355
32	2261.85	.76	2	1889.01	.91	32	1621.70	1.06	2	1420.66	1.21	32	1263.97	1.36
33	2247.06	.765	3	1878.69	.915	33	1614.09	1.065	3	1414.82	1.215	33	1259.34	1.365
34	2232.47	.77	4	1868.48	.92	34	1606.54	1.07	4	1409.01	1.22	34	1254 74	1.37
35	2218.06	.775	5	1858.38	.925	35	1599.07	1.075	5	1403.26	1.225	35	1250.18	1.375
36	2203.85	.78	6	1848.39	.93	36	1591.67	1.08	6	1397.56	1.23	36	1245.66	1.38
37	2189.82	.785	7	1838.50	.935	37	1584.34	1.085	7	1391.90	1.235	37	1241.16	1.385
38	2175.95	.79	8	1828.72	.94	38	1577.07	1.09	8	1386.29	1.24	38	1236.70	1.39
39	2162.27	.795	9	1819 05	.945	39	1569.87	1.095	9	1380.72	1 245	39	1232.26	1.395
40	2148.75	.8	10	1809.48	.95	40	1562.73	1.1	10	1375.20	1.25	40	1227.86	1.4
41	2135.41	.805	11	1800.00	.955	41	1555.66	1.105	11	1369.72	1.255	41	1223.49	1.405
42	2122.22	.81	12	1790.62	.96	42	1548.65	1.11	12	1364.29	1.26	42	1219.15	1.41
43	2109.20	.815	13	1781.35	.965	43	1541.71	1.115	13	1358.90	1.265	43	1215.05	1.415
44	2096.34	.82	14	1772.17	.97	44	1534.82	1.12	14	1353.55	1.27	44	1210.56	1.42
45	2083.63	.825	15	1763.08	.975	45	1528.00	1.125	15	1348.23	1.275	45	1206.31	1.425
46	2071.09	.83	16	1754.08	.98	46	1521.24	1.13	16	1342.93	1.28	46	1202.10	1.43
47	2058.69	.835	17	1745.15	.985	47	1514.54	1.135	17	1337.74	1.285	47	1197.91	1.435
48	2046.43	.84	18	1736.36	.99	48	1507.90	1.14	18	1332.56	1.29	48	1193.75	1.44
49	2034.32	.845	19	1727.63	.995	49	1501.32	1.145	19	1327.42	1.295	49	1189.62	1.445
50	2022.35	.85	20	1719.00	1.	50	1494.78	1.15	20	1322.31	1.3	50	1185.52	1.45
51	2010.53	.855	21	1710.45	1.005	51	1488.32	1.155	21	1317.24	1.305	51	1181.45	1.455
52	1998.84	.86	22	1701.98	1.01	52	1481.90	1.16	22	1312.21	1.31	52	1177.40	1.46
53	1987.29	.865	23	1693.60	1.015	53	1475.54	1.165	23	1307.23	1.315	53	1173.38	1 465
54	1975.86	.87	24	1685.30	1.02	54	1469.23	1.17	24	1302.27	1.32	54	1169.39	1.47
55	1964.57	.875	25	1677.08	1.025	55	1462.98	1.175	25	1297.35	1.325	55	1165.43	1.475
56	1953.41	.88	26	1668.93	1.03	56	1456.78	1.18	26	1292.48	1.33	56	1161.48	1.48
57	1942.37	.885	27	1660.87	1.035	57	1450 63	1.185	27	1287.64	1.335	57	1157 57	1.485
58	1931.46	.89	28	1652.88	1.04	58	1444.54	1.19	28	1282.84	1.34	58	1153.69	1.49
59	1920.67	.895	29	1644.98	1.045	59	1438.50	1.195	29	1278.07	1.345	59	1149.83	1.495
60	1910.00	.9	30	1637.15	1.05	60	1432.50	1.2	30	1273.33	1.35	60	1146.00	1.5

TABLE V.—RADII, AND DEFLECTION ANGLES IN MINUTES PER FOOT OF LENGTH.

Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.
5° 0′	1146.00	1.5	5° 30′	1041.82	1.65	6° 0′	955.00	1.8	6° 30′	881.54	1.95	7° 0′	818.37	2.1
1	1142.19	1.505	31	1038.68	1.655	1	952.35	1.805	31	879.28	1.955	1	816.62	2.105
2	1138.41	1.51	32	1035.54	1.66	2	949.73	1.81	32	877.04	1.96	2	814.69	2.11
3	1134.66	1 515	33	1032.43	1.665	3	947.11	1.815	33	874.80	1.965	3	812.78	2.115
4	1130.92	1.52	34	1029.34	1.67	4	944.51	1.82	34	872.58	1.97	4	810.85	2.12
5	1127.21	1.525	35	1026.27	1.675	5	941.92	1.825	35	870.38	1.975	5	808.94	2.125
6	1123.53	1.53	36	1023.21	1.68	6	939.35	1.83	36	868.18	1.98	6	807.04	2.13
7	1119.87	1.535	37	1020.18	1.685	7	936.79	1.835	37	865.99	1.985	7	805.15	2.135
8	1116.23	1.54	38	1017.16	1.69	8	934.24	1.84	38	863.82	1.99	8	803.27	2.14
9	1112.62	1.545	39	1014.16	1.695	9	931.71	1.845	39	861.66	1.995	9	801.40	2.145
10	1109.03	1.55	40	1011.17	1.7	10	929.19	1.85	40	859.50	2.	10	799.54	2.15
11	1105.46	1.555	41	1008.21	1.705	11	926.69	1.855	41	857.36	2.005	11	797.68	2.155
12	1101.92	1.56	42	1005.26	1.71	12	924.20	1.86	42	855.22	2.01	12	795.84	2.16
13	1098.40	1.565	43	1002.33	1.715	13	921.72	1.865	43	853.10	2.015	13	794.00	2.165
14	1094.91	1.57	44	999.42	1.72	14	919.25	1.87	44	850.99	2.02	14	792.17	2.17
15	1091.43	1.575	45	996.53	1.725	15	916.80	1.875	45	848.89	2.025	15	790.35	2.175
16	1087.97	1.58	46	993.65	1.73	16	914.36	1.88	46	846.80	2.03	16	788.53	2.18
17	1084.54	1.585	47	990.79	1.735	17	911.94	1.885	47	844.72	2.035	17	786.74	2.185
18	1081.13	1 59	48	987.93	1.74	18	909.52	1.89	48	842 65	2.04	18	784.93	2.19
19	1077.74	1.595	49	985.10	1.745	19	907.13	1.895	49	840.59	2.045	19	783.14	2.195
20	1074.38	1.60	50	982.29	1.75	20	904.74	1.9	50	838.54	2.05	20	781.36	2.2
21	1071.03	1.605	51	979.49	1.755	21	902.36	1.905	51	836.50	2.055	21	779.59	2.205
22	1067.70	1.61	52	976.70	1.76	22	900.00	1.91	52	834.47	2.06	22	777.83	2.21
23	1064.40	1.615	53	973.94	1.765	23	897.65	1.915	53	832.44	2.065	23	776.07	2.215
24	1061.11	1.62	54	971.19	1.77	24	895.31	1.92	54	830.43	2.07	24	774.33	2.22
25	1057.84	1.625	55	968.45	1.775	25	892.99	1.925	55	828.43	2.075	25	772.58	2.225
26	1054.60	1.63	56	965.73	1.78	26	890.67	1.93	56	826.44	2.08	26	770.85	2.23
27	1051.38	1.635	57	963.03	1.785	27	888.37	1.935	57	824.46	2.085	27	769.13	2.235
28	1048.17	1.64	58	960.34	1.79	28	886.08	1.94	58	822.49	2.09	28	767.41	2.24
29	1044.99	1.645	59	957.65	1.795	29	883.80	1.945	59	820.53	2.095	29	765.70	2.245
30	1041.82	1.65	60	955.00	1.8	30	881.54	1.95	60	818.57	2.1	30	764.00	2.25

TABLE V.—RADII, AND DEFLECTION ANGLES IN MINUTES PER FOOT OF LENGTH.

Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.	Deg. *D*	Radius *R*	Def. per Foot.
7° 30′	764.00	2.25	8° 0′	716.25	2.4	9° 0′	636.67	2.7	10° 0′	573.00	3.	11° 0	520.91	3.3
31	762.31	2.255	2	713 28	2.41	2	634.32	2.71	2	571.10	3.01	2	519.34	3.31
32	760.62	2.26	4	710.33	2.42	4	631.99	2.72	4	569.20	3.02	4	517.77	3.32
33	758.94	2.265	6	707.41	2.43	6	629.66	2.73	6	567.33	3.03	6	516.22	3.33
34	757.27	2.27	8	704.51	2.44	8	627.37	2.74	8	565.46	3.04	8	514.67	3.34
35	755.60	2.275	10	701.63	2.45	10	625.09	2.75	10	563.60	3.05	10	513.13	3.35
36	753.95	2.28	12	698.78	2.46	12	622.83	2.76	12	561.76	3.06	12	511.61	3.36
37	752.30	2.285	14	695.95	2.47	14	620.58	2.77	14	559.93	3.07	14	510.09	3.37
38	750.66	2.29	16	693.14	2.48	16	618.35	2.78	16	558.12	3.08	16	508.58	3.38
39	749.02	2.295	18	690.36	2.49	18	616.13	2.79	18	556.31	3.09	18	507.08	3.39
40	747.39	2.3	20	687.60	2.5	20	613.91	2.8	20	554.52	3.1	20	505.59	3.4
41	745 77	2.305	22	684.86	2.51	22	611.74	2.81	22	552.73	3.11	22	504.11	3.41
42	744.16	2.31	24	682.14	2.52	24	609.57	2.82	24	550.96	3.12	24	502.63	3.42
43	742.55	2.315	26	679.45	2.53	26	607.42	2.83	26	549.20	3.13	26	501.17	3.43
44	740.95	2.32	28	676.77	2.54	28	605.28	2.84	28	547.45	3.14	28	499.71	3.44
45	739.36	2.325	30	674.12	2.55	30	603.16	2.85	30	545.72	3.15	30	498.26	3.45
46	737.77	2.33	32	671.48	2.56	32	601.05	2.86	32	543.99	3.16	32	496.82	3.46
47	736.19	2.335	34	668.87	2.57	34	598.95	2.87	34	542.27	3.17	34	495.39	3.47
48	734.62	2.34	36	666.28	2 58	36	596 88	2.88	36	540.56	3.18	36	493.97	3 48
49	733.05	2.345	38	663.71	2.59	38	594 81	2.89	38	538.87	3.19	38	492.55	3.49
50	731.49	2.35	40	661.15	2.6	40	592.76	2.9	40	537.19	3.2	40	491.14	3.5
51	729.94	2 355	42	658.62	2.61	42	590.72	2.91	42	535.52	3.21	42	489.74	3.51
52	728.39	2 36	44	656.11	2.62	44	588.70	2.92	44	533.85	3.22	44	488.35	3.52
53	726.85	2.365	46	653.61	2.63	46	586.69	2.93	46	532.20	3.23	46	486.97	3.53
54	725.32	2.37	48	651.14	2.64	48	584.69	2.94	48	530.56	3.24	48	485.59	3 54
55	723.79	2.375	50	648.68	2.65	50	582.71	2.95	50	528.92	3.25	50	484.23	3.55
56	722.27	2.38	52	646.24	2.66	52	580.74	2.96	52	527.30	3.26	52	482.87	3.56
57	720.76	2.385	54	643.82	2.67	54	578.79	2.97	54	525.69	3.27	54	481.51	3 57
58	719.25	2 39	56	641.42	2.68	56	576.85	2.98	56	524.09	3.28	56	480.17	3.58
59	717.75	2.395	58	639.04	2 69	58	574.92	2 99	58	522.49	3.29	58	478.83	3.59
60	716.25	2.4	60	636.67	2.7	60	573.00	3.	60	520.91	3.3	60	477.50	3.6

TABLE V.—RADII, AND DEFLECTION ANGLES IN MINUTES PER FOOT OF LENGTH.

Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.
12° 0′	477.50	3.6	13° 0′	440.77	3.9	14° 0′	409.29	4.2	15° 0′	382.00	4.5	16° 0′	358.12	4.8	21° 0′	272.86	6.3
2	476.18	3.61	2	439.64	3.91	2	408.31	4.21	2	381.15	4.51	10	354.43	4.85	10	270.71	6.35
4	474.86	3.62	4	438.52	3.92	4	407.35	4.22	4	380.31	4.52	20	350 82	4.9	20	268.59	6.4
6	473.56	3.63	6	437.40	3.93	6	406.39	4.23	6	379.47	4.53	30	347.27	4.95	30	266.51	6.45
8	472.25	3.64	8	436.29	3.94	8	405.43	4.24	8	378.64	4.54	40	343.80	5.	40	264.46	6.5
10	470.96	3.65	10	435.19	3.95	10	404.47	4.25	10	377.80	4.55	50	340.40	5.05	50	262.44	6.55
12	469.67	3.66	12	434.09	3.96	12	403.52	4.26	12	376.97	4.56	17 0	337.06	5.1	22 0	260.45	6.6
14	468.39	3.67	14	433.00	3.97	14	402 58	4.27	14	376.15	4.57	10	333.79	5.15	10	258.50	6.65
16	467.12	3.68	16	431.91	3.98	16	401.64	4.28	16	375.33	4.58	20	330.58	5.2	20	256.57	6.7
18	465.85	3.69	18	430.83	3 99	18	400.70	4.29	18	374.51	4.59	30	327.44	5 25	30	254.67	6.75
20	464.60	3.7	20	429.75	4.	20	399.77	4.3	20	373.69	4.6	40	324.34	5.3	40	252.79	6.8
22	463.34	3.71	22	428.68	4.01	22	398.84	4.31	22	372.89	4.61	50	321.31	5.35	50	250.95	6.85
24	462.10	3.72	24	427.61	4.02	24	397.92	4.32	24	372.08	4.62	18 0	318.33	5.4	23 0	249.13	6.9
26	460.86	3.73	26	426.55	4.03	26	397.00	4 33	26	371.28	4.63	10	315.41	5.45	10	247.34	6.95
28	459.63	3.74	28	425.49	4.04	28	396.08	4.34	28	370.47	4.64	20	312.55	5.5	20	245.57	7.
30	458.40	3.75	30	424.45	4.05	30	395.17	4.35	30	369.68	4.65	30	309.73	5.55	30	243.83	7.05
32	457.18	3.76	32	423.40	4.06	32	394.27	4.36	32	368.89	4.66	40	306.96	5.6	40	242.11	7.1
34	455.97	3.77	34	422.36	4.07	34	393.37	4.37	34	368.09	4.67	50	304.24	5.65	50	240 42	7.15
36	454.76	3.78	36	421.32	4.08	36	392.46	4.38	36	367.31	4.68	19 0	301.58	5.7	24 0	238.75	7.2
38	453.56	3.79	38	420 30	4.09	38	391.57	4.39	38	366.53	4.69	10	298.96	5.75	10	237.10	7.25
40	452.37	3.8	40	419.27	4.1	40	390.68	4.4	40	365.75	4.7	20	296.38	5.8	20	235.48	7.3
42	451.18	3.81	42	418.25	4.11	42	389.80	4.41	42	364.97	4.71	30	293.85	5.85	30	233.88	7.35
44	450.00	3.82	44	417.23	4.12	44	388 92	4.42	44	364.20	4.72	40	291.36	5.9	40	232.30	7.4
46	448.82	3.83	46	416.22	4.13	46	388.04	4.43	46	363.43	4.73	50	288.91	5.95	50	230.74	7.45
48	447.66	3.84	48	415.22	4.14	48	387.16	4.44	48	362.66	4.74	20 0	286.50	6.	25 0	229.20	7.5
50	446.50	3.85	50	414.21	4.15	50	386.29	4.45	50	361.90	4.75	10	284.13	6.05	10	227.68	7.55
52	445.34	3.86	52	413.22	4.16	52	385.43	4.46	52	361.13	4.76	20	281.80	6.1	20	226.18	7.6
54	444.19	3.87	54	412.23	4.17	54	384.57	4.47	54	360.38	4 77	30	279.52	6.15	30	224.71	7.65
56	443.04	3 88	56	411.24	4.18	56	383.70	4.48	56	359.62	4.78	40	277.26	6.2	40	223.25	7.7
58	441.90	3.89	58	410.26	4.19	58	382.85	4.49	58	358.87	4.79	50	275.04	6.25	50	221.81	7.75
60	440.77	3.9	60	409.29	4.2	60	382.00	4.5	60	358.12	4.8	60	272.86	6.3	60	220.38	7.8

TABLE V.—RADII AND DEFLECTION ANGLES IN MINUTES PER FOOT OF LENGTH.

Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.	Deg. D	Radius R	Def. per Foot.
26° 0′	220.38	7.8	31° 0′	184.84	9.3	41° 0′	139.75	12.3	51° 0′	112.35	15.3	61° 0′	93.94	18.3	76°	75.40	22.8
10	218.98	7.85	20	182.87	9.4	20	138.63	12.4	20	111.63	15.4	30	93.17	18.45	77	74.41	23.1
20	217.60	7 9	40	180.95	9.5	40	137.52	12.5	40	110.90	15.5	62	92.42	18.6	78	73.46	23.4
30	216.23	7.95	32	179.06	9.6	42	136.43	12.6	52	110.19	15.6	30	91.68	18.75	79	72.53	23.7
40	214.88	8.	20	177.22	9.7	20	135.35	12 7	20	109.49	15.7	63	90.95	18.9	80	71.62	24.0
50	213 54	8.05	40	175.41	9.8	40	134.30	12.8	40	108.80	15.8	30	90.24	19.05	81	70.74	24.3
27	212.22	8.1	33	173.64	9.9	43	133.25	12.9	53	108.11	15.9	64	89.53	19.2	82	69.88	24.6
10	210.92	8.15	20	171.90	10.0	20	132.23	13.0	20	107.43	16.0	30	88.84	19.35	83	69.03	24.9
20	209.63	8.2	40	170.20	10.1	40	131.22	13.1	40	106.77	16.1	65	88.15	19.5	84	68.21	25.2
30	208.36	8.25	34	168.53	10.2	44	130.23	13.2	54	106.11	16.2	30	87.48	19.65	85	67.41	25.5
40	207.11	8.3	20	166.89	10.3	20	129.25	13.3	20	105.46	16.3	66	86.82	19.8	86	66.63	25.8
50	205.87	8.35	40	165.28	10.4	40	128.28	13.4	40	104.82	16.4	30	86.16	19.95	87	65.86	26.1
28	204.64	8.4	35	163.71	10.5	45	127.33	13.5	55	104.18	16 5	67	85.52	20.1	88	65.11	26.4
10	203.43	8.45	20	162.17	10.6	20	126.40	13.6	20	103.55	16.6	30	84.89	20.25	89	64.38	26.7
20	202 24	8.5	40	160.67	10.7	40	125.47	13.7	40	102.93	16.7	68	84.26	20.4	90	63 67	27.0
30	201.05	8.55	36	159.17	10.8	46	124.56	13.8	56	102.32	16.8	30	83.65	20.55	91	62.97	27.3
40	199.88	8.6	20	157.70	10.9	20	123.67	13.9	20	101.71	16.9	69	83.04	20.7	92	62.28	27.6
50	198.73	8 65	40	156.27	11.0	40	122.79	14.0	40	101.12	17.0	30	82.45	20.85	93	61.61	27 9
29	197.59	8.7	37	154.86	11.1	47	121.92	14.1	57	100.53	17.1	70	81.86	21.0	94	60.96	28.2
10	196.46	8.75	20	153 48	11.2	20	121.05	14.2	20	99.94	17.2	30	81.28	21.15	95	60.32	28.5
20	195.34	8.8	40	152.12	11.3	40	120.21	14.3	40	99.36	17.3	71	80.70	21.3	96	59.69	28.8
30	194.24	8.85	38	150.79	11.4	48	119.37	14.4	58	98.79	17.4	30	80.14	21.45	97	59.07	29.1
40	193.15	8.9	20	149.48	11.5	20	118.55	14 5	20	98.23	17.5	72	79.58	21.6	98	58.47	29.4
50	192 07	8.95	40	148.19	11.6	40	117.74	14.6	40	97.67	17.6	30	79.04	21.75	99	57.88	29.7
30	191.00	9.	39	146.92	11.7	49	116.94	14.7	59	97.12	17.7	73	78.49	21.9	100	57.30	30.0
10	189.91	9.05	20	145.68	11.8	20	116.15	14.8	20	96.57	17.8	30	77.96	22.05	101	56.73	30.3
20	188.90	9.1	40	144.45	11.9	40	115.37	14.9	40	96.03	17.9	74	77.43	22.2	102	56.18	30.6
30	187.87	9.15	40	143.24	12.0	50	114.60	15 0	60	95 50	18.0	30	76.91	22.35	103	55 63	30.9
40	186.85	9.2	20	142.06	12.1	20	113.84	15.1	20	94.97	18.1	75	76.40	22.5	104	55.10	31.2
50	185.84	9.25	40	140.90	12.2	40	113.09	15.2	40	94.46	18.2	30	75.89	22 65	105	54.57	31.5
60	184.84	9.3	60	139.75	12.3	60	112.35	15.3	60	93.94	18.3	60	75.40	22.8	106	54.05	31.8

TABLE V.—*Continued.*

Deg. *D*	Radius *R*	Def. per Foot.
106°	54.05	31.8
107	53.55	32.1
108	53.06	32.4
109	52.57	32.7
110	52.09	33.0
111	51.62	33.3
112	51.16	33.6
113	50.71	33.9
114	50.26	34.2
115	49.83	34.5
116	49.40	34.8
117	48.97	35.1
118	48.56	35.4
119	48.15	35.7
120	47.75	36.0
121	47.36	36.3
122	46.97	36.6
123	46.59	36.9
124	46.21	37.2
125	45.84	37.5
126	45.47	37.8
127	45.12	38.1
128	44.76	38.4
129	44.42	38 7
130	44.08	39.0
131	43.74	39.3
132	43.41	39.6
133	43.09	39.9
134	42.76	40.2
135	42.44	40.5
136	42.13	40.8

TABLE VI.—ARC EXCESS *

Deg. *D*	Actual Arc. One Sta.
0°	100.000
1	100.001
2	100.005
3	100.011
4	100.020
5	100.032
6	100.046
7	100.062
8	100.020
9	100.026
10	100.032
11	100.038
12	100.046
13	100.054
14	100.062
15	100.071
16	100.020
17	100 023
18	100.026
19	100.029
20	100.032
21	100.035
22	100.038
23	100.042
24	100.046
25	100.050
26	100.054
27	100.058
28	100 062
29	100.066
30	100.071

TABLE VII.—TANGENTS AND EXTERNALS FOR A 1° CURVE.

Angle Δ	Tangent *T*	External *E*	Angle Δ	Tangent *T*	External *E*
1°	50.00	.216	6°	300.29	7.863
10′	58.34	.296	10′	308.65	8.308
20	66.67	.386	20	317.01	8.762
30	75.01	.490	30	325.37	9.230
40	83.34	.605	40	333.73	9.710
50	91.68	.733	50	342.09	10.203
2	100.02	.873	7	350.46	10.707
10	108.35	1.024	10	358 82	11.224
20	116.69	1.187	20	367.19	11.753
30	125.03	1.364	30	375.56	12.295
40	133.37	1.551	40	383.93	12.847
50	141.70	1.751	50	392.30	13.413
3	150.04	1.963	8	400.68	13.992
10	158.38	2.187	10	409.05	14 583
20	166.73	2.425	20	417.43	15.185
30	175.06	2.674	30	425 81	15.800
40	183.41	2.934	40	434.19	16.427
50	191.75	3.207	50	442.57	17.067
4	200.09	3.492	9	450.96	17.719
10	208.44	3.790	10	459.35	18.383
20	216.78	4.099	20	467.73	19.059
30	225.13	4.420	30	476.12	19.748
40	233.47	4.754	40	484.52	20.447
50	241.82	5.100	50	492.91	21.162
5	250.17	5.458	10	501.31	21.887
10	258.53	5.829	10	509.71	22.625
20	266.87	6.211	20	518.11	23.376
30	275.23	6.605	30	526.51	24.139
40	283.58	7.013	40	534.92	24.913
50	291.94	7.433	50	543.32	25.703

* See page 39.

TABLE VII.—TANGENTS AND EXTERNALS FOR A 1° CURVE.

Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E
11°	551.74	26.501	17°	856.35	63.638	23°	1165.8	117.39	29°	1481.9	188.52
10′	560.15	27.313	10′	864.87	64.904	10′	1174.5	119.13	10′	1490 8	190 75
20	568.56	28.138	20	873.40	66.182	20	1183.1	120.88	20	1499.7	193.00
30	576.98	28.976	30	881.93	67.474	30	1191.8	122 64	30	1508.6	195.26
40	585 40	29.825	40	890.46	68.780	40	1200.5	124.42	40	1517.5	197.54
50	593.82	30.687	50	899.00	70.096	50	1209.2	126.21	50	1526.4	199.83
12	602.25	31.562	18	907.54	71.425	24	1217 9	128.01	30	1535.4	202.13
10	610.67	32.449	10	916.08	72.768	10	1226.7	129.83	10	1544.3	204.45
20	619.10	33.349	20	924.63	74.123	20	1235.4	131.66	20	1553.2	206.79
30	627.53	34.261	30	933.18	75.492	30	1244 1	133.50	30	1562.2	209.13
40	635.97	35.184	40	941.74	76.873	40	1252.8	135.36	40	1571.1	211.49
50	644.41	36.121	50	950.30	78.268	50	1261.6	137.24	50	1580.1	213.87
13	652.85	37.070	19	958.87	79.676	25	1270.3	139.12	31	1589.1	216.26
10	661.29	38.033	10	967.44	81 097	10	1279.1	141.02	10	1598.0	218.67
20	669.74	39.008	20	976.01	82.530	20	1287.8	142.94	20	1607.0	221.09
30	678.19	39.995	30	984.59	83.977	30	1296.6	144.86	30	1616.0	223 52
40	686.64	40.995	40	993 18	85.436	40	1305.3	146.80	40	1625.0	225.97
50	695.09	42.007	50	1001.8	86.908	50	1314.1	148.76	50	1634.0	228.44
14	703.55	43.031	20	1010.4	88.395	26	1322.9	150.72	32	1643.1	230.92
10	712.02	44.068	10	1018.9	89.893	10	1331.7	152.70	10	1652.1	233 41
20	720.48	45.119	20	1027.5	91.405	20	1340.4	154 70	20	1661.1	235.92
30	728.95	46.181	30	1036.1	92.930	30	1349.2	156.71	30	1670.1	238.44
40	737.42	47.256	40	1044.8	94.468	40	1358.0	158.73	40	1679.2	240.98
50	745.89	48.343	50	1053.4	96.019	50	1366.8	160.77	50	1688.2	243.53
15	754.37	49.443	21	1062.0	97.584	27	1375.7	162.82	33	1697.3	246.09
10	762.85	50.557	10	1070.6	99.161	10	1384.5	164.88	10	1706.4	248.68
20	771.33	51.683	20	1079.2	100.75	20	1393.3	166.96	20	1715.4	251.28
30	779.82	52.820	30	1087.9	102.35	30	1402.1	169.06	30	1724.5	253.89
40	788.30	53.972	40	1096.5	103.97	40	1411 0	171.16	40	1733.6	256.52
50	796.80	55.135	50	1105.2	105.60	50	1419.8	173.28	50	1742.7	259.16
16	805.29	56.312	22	1113.8	107.24	28	1428.6	175.42	34	1751.8	261.81
10	813.80	57.501	10	1122.4	108.90	10	1437.5	177.56	10	1761.0	264.49
20	822.30	58.703	20	1131.1	110.57	20	1446.4	179.73	20	1770.1	267.17
30	830.81	59.917	30	1139.8	112.26	30	1455.2	181.90	30	1779.2	269.87
40	839.32	61.145	40	1148.4	113.95	40	1464.1	184.09	40	1788.4	272.59
50	847.83	62.386	50	1157.1	115.66	50	1473.0	186.30	50	1797.5	275.32

TABLE VII.—TANGENTS AND EXTERNALS FOR A 1° CURVE.

Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E
35°	1806 7	278 07	41°	2142.4	387.40	47°	2491.5	518.23	53°	2856.9	672.70
10′	1815.8	280.83	10′	2151.9	390.74	10′	2501.4	522.19	10′	2867.3	677.36
20	1825.0	283 61	20	2161.4	394.09	20	2511.3	526.17	20	2877.7	682.03
30	1834.2	286.41	30	2170.9	397.46	30	2521.3	530.16	30	2888.2	686.73
40	1843.4	289.21	40	2180.5	400.84	40	2531.2	534.18	40	2898.6	691.44
50	1852.6	292.04	50	2190.0	404.24	50	2541.2	538.21	50	2909.1	696.17
36	1861.8	294.88	42	2199.5	407.67	48	2551.2	542.27	54	2919.6	700.93
10	1871 0	297.73	10	2209.1	411.10	10	2561.2	546.34	10	2930.1	705.70
20	1880.2	300.60	20	2218.7	414.55	20	2571.2	550.43	20	2940.6	710.50
30	1889.5	303.49	30	2228.3	418.02	30	2581.2	554.54	30	2951.2	715 32
40	1898.7	306.39	40	2237.8	421.50	40	2591.2	558.66	40	2961.7	720.16
50	1908.0	309.31	50	2247.5	425.00	50	2601.2	562.81	50	2972.3	725.02
37	1917.2	312.24	43	2257.1	428.53	49	2611.3	566.97	55	2982.9	729.90
10	1926.5	315 19	10	2266.7	432.06	10	2621.4	571.15	10	2993.5	734.80
20	1935.8	318.15	20	2276 4	435 62	20	2631.5	575.36	20	3004.1	739.72
30	1945.1	321.14	30	2286.1	439.19	30	2641.6	579.58	30	3014.7	744.67
40	1954.4	324.13	40	2295.7	442.77	40	2651.7	583.82	40	3025.4	749.63
50	1963.7	327.14	50	2305.4	446.38	50	2661.8	588.08	50	3036.0	754.62
38	1973 0	330.17	44	2315.1	449.99	50	2672.0	592.36	56	3046.7	759.63
10	1982.3	333.21	10	2324.8	453.64	10	2682.1	596.65	10	3057.4	764.66
20	1991.7	336.27	20	2334.5	457.30	20	2692.2	600 97	20	3068.1	769.71
30	2001.0	339 35	30	2344.2	460.98	30	2702.4	605.31	30	3078.8	774.78
40	2010.4	342.43	40	2353.9	464.67	40	2712.6	609.66	40	3089.6	779.87
50	2019.7	345.54	50	2363.7	468.38	50	2722.9	614.04	50	3100.4	784.99
39	2029.1	348.66	45	2373.4	472.11	51	2733.1	618.43	57	3111.1	790.13
10	2038.5	351.80	10	2383.2	475.85	10	2743.3	622.84	10	3121.9	795.29
20	2047.9	354 96	20	2393.0	479.62	20	2753.6	627.28	20	3132.8	800.47
30	2057.3	358.13	30	2402.8	483.40	30	2763.8	631.73	30	3143.6	805.67
40	2066 7	361.32	40	2412.6	487.19	40	2774.1	636.20	40	3154.4	810.90
50	2076.1	364.52	50	2422.4	491.01	50	2784.4	640.70	50	3165.3	816.15
40	2085 6	367.74	46	2432.2	494.84	52	2794.7	645.21	58	3176.2	821.42
10	2095.0	370.97	10	2442 1	498.69	10	2805.0	649.74	10	3187 1	826.71
20	2104.4	374.23	20	2451.9	502.57	20	2815.4	654.29	20	3198.0	832.03
30	2113.9	377.50	30	2461.8	506.45	30	2825.7	658.87	30	3209.0	837.37
40	2123.4	380.78	40	2471.7	510.36	40	2836.1	663.46	40	3219.9	842.73
50	2132.9	384.08	50	2481.6	514.28	50	2846.5	668.07	50	3230.9	848.11

TABLE VII.—TANGENTS AND EXTERNALS FOR A 1° CURVE.

Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E	Angle Δ	Tang. T	Extern. E
59°	3241.9	853.52	65°	3650.4	1064.0	71°	4087.2	1308.3	77°	4557.9	1591.7
10′	3252.9	858.94	10′	3662.1	1070.3	10′	4099.7	1315.6	10′	4571.5	1600.2
20	3263.9	864.40	20	3673.9	1076.6	20	4112.4	1323.0	20	4585.1	1608.7
30	3275.0	869.87	30	3685.7	1083.0	30	4125.0	1330.4	30	4598.8	1617.3
40	3286.0	875.35	40	3697.5	1089.4	40	4137.7	1337.8	40	4612.5	1625.8
50	3297.1	880.88	50	3709.3	1095.8	50	4150.4	1345.2	50	4626.3	1634.5
60	3308.2	886.43	66	3721.1	1102.3	72	4163.1	1352.7	78	4640.1	1643.1
10	3319.3	892.00	10	3733.0	1108.7	10	4175.8	1360.2	10	4653.9	1651.9
20	3330.5	897.59	20	3744.9	1115.2	20	4188.6	1367.7	20	4667.7	1660.6
30	3341.6	903.21	30	3756.8	1121.7	30	4201.4	1375.3	30	4681.6	1669.4
40	3352.8	908.85	40	3768.7	1128.3	40	4214.2	1382.9	40	4695.5	1678.2
50	3364.0	914.51	50	3780.6	1134.9	50	4227.1	1390.5	50	4709.5	1687.0
61	3375.2	920.19	67	3792.6	1141.5	73	4240.0	1398.1	79	4723.4	1695.9
10	3386.5	925.90	10	3804.6	1148.1	10	4252.9	1405.8	10	4737.5	1704.8
20	3397.7	931.63	20	3816.6	1154.7	20	4265.8	1413.6	20	4751.5	1713.8
30	3409.0	937.39	30	3828.7	1161.4	30	4278.8	1421.3	30	4765.6	1722.8
40	3420.3	943.17	40	3840.7	1168.1	40	4291.8	1429.1	40	4779.7	1731.8
50	3431.6	948.98	50	3852.8	1174.9	50	4304.8	1436.9	50	4793.9	1740.9
62	3442.9	954.81	68	3864.9	1181.6	74	4317.9	1444.7	80	4808.1	1750.0
10	3454.3	960.66	10	3877.1	1188.4	10	4330.9	1452.6	10	4822.3	1759.2
20	3465.7	966.54	20	3889.2	1195.3	20	4344.1	1460.5	20	4836.5	1768.3
30	3477.1	972.45	30	3901.4	1202.1	30	4357.2	1468.5	30	4850.8	1777.6
40	3488.5	978.37	40	3913.6	1209.0	40	4370 4	1476.5	40	4865.1	1786.8
50	3499.9	984.33	50	3925.9	1215.9	50	4383.6	1484.5	50	4879.5	1796.1
63	3511.4	990.30	69	3938.1	1222.8	75	4396.8	1492.5	81	4893.9	1805.5
10	3522.8	996.30	10	3950.4	1229.8	10	4410.0	1500.5	10	4908.3	1814.8
20	3534.3	1002.3	20	3962.7	1236.8	20	4423.3	1508.7	20	4922.8	1824.3
30	3545.8	1008.4	30	3975.0	1243.8	30	4436.7	1516.9	30	4937.3	1833.7
40	3557.4	1014.5	40	3987.4	1250.9	40	4450.0	1525.0	40	4951.8	1843.2
50	3568.9	1020.6	50	3999.8	1257.9	50	4463.4	1533.2	50	4966.4	1852.8
64	3580.5	1026.7	70	4012.2	1265.0	76	4476.8	1541.5	82	4981.0	1862.3
10	3592.1	1032.9	10	4024.6	1272.2	10	4490.2	1549.8	10	4995.7	1872.0
20	3603.7	1039 0	20	4037.1	1279.4	20	4503.7	1558.1	20	5010.4	1881.6
30	3615.4	1045.2	30	4049.6	1286.6	30	4517.2	1566.4	30	5025.1	1891.3
40	3627.0	1051.5	40	4062.1	1293.8	40	4530.7	1574.8	40	5039.8	1901.1
50	3638.7	1057.7	50	4074.6	1301.0	50	4544.3	1583.2	50	5054.6	1910.8

TABLE VII.—TANGENTS AND EXTERNALS FOR A 1° CURVE.

Angle Δ	Tangent T	External E	Angle Δ	Tangent T	External E	Angle Δ	Tangent T	External E	Angle Δ	Tangent T	External E
83°	5069.5	1920.7	88°	5533.4	2235.7	93°	6038.2	2591.2	98°	6591.6	3004.0
10′	5084.4	1930.5	10′	5549.5	2246.9	10′	6055 8	2607.0	10′	6611.0	3018.6
20	5099.3	1940 4	20	5565.7	2258.1	20	6073.4	2619.8	20	6630.5	3033.3
30	5114.2	1950.4	30	5581.9	2269.4	30	6091.2	2632.7	30	6650.0	3048.1
40	5129.2	1960.4	40	5598.2	2280.8	40	6109.0	2645.7	40	6669.6	3063.0
50	5144.3	1970.4	50	5614 4	2292.2	50	6126.8	2658.7	50	6689.2	3077.9
84	5159.3	1980.5	89	5630.9	2303.7	94	6144.7	2671.8	99	6709.0	3092.9
10	5174.4	1990.6	10	5647.3	2315.2	10	6162.6	2684.9	10	6728.8	3107.9
20	5189.6	2000.8	20	5663.7	2326.7	20	6180.6	2698.1	20	6748.6	3123.1
30	5204.8	2011.0	30	5680.2	2338.3	30	6198.7	2711.4	30	6768.6	3138.3
40	5220.0	2021.2	40	5696.8	2350.0	40	6216.8	2724 7	40	6788.6	3153.5
50	5235.3	2031.5	50	5713.4	2361.7	50	6235.0	2738.0	50	6808.6	3168.9
85	5250.6	2041.9	90	5730.0	2373.5	95	6253 2	2751.5	100	6828.7	3184.3
10	5265 9	2052.2	10	5746.7	2385.3	10	6271.5	2765.0	10	6849.0	3199.8
20	5281.3	2062.7	20	5763.4	2397.1	20	6289.8	2778.5	20	6869.2	3215.4
30	5296.8	2073.1	30	5780.2	2409.0	30	6308.2	2792 1	30	6889.6	3231.0
40	5312 2	2083.6	40	5797.1	2421.0	40	6326.7	2805.8	40	6910.0	3246.7
50	5327.8	2094.2	50	5814.0	2463.0	50	6345.2	2819.6	50	6930.5	3262.5
86	5343.3	2104.8	91	5830 9	2445.1	96	6363.8	2833.4	101	6951.1	3278.3
10	5358.9	2115.4	10	5847.8	2457.2	10	6382.5	2847.2	10	6971 7	3294.3
20	5374.6	2126.1	20	5864.9	2469.4	20	6401.2	2861.1	20	6992.4	3310.3
30	5390.3	2136.9	30	5882.0	2481.6	30	6419.9	2875.1	30	7013.2	3326.4
40	5406.0	2147.7	40	5899.2	2493.9	40	6438.8	2889.2	40	7034.0	3342.5
50	5421.8	2158.5	50	5916.4	2506.3	50	6457.6	2903.3	50	7055.0	3358.7
87	5437.6	2169.4	92	5933.6	2518.6	97	6476.6	2917.5	102	7076.0	3375.1
10	5453.4	2180.3	10	5950.9	2531.1	10	6495.6	2931.7	10	7097.0	3391.5
20	5469 3	2191.3	20	5968.2	2543.6	20	6514 7	2946.1	20	7118.2	3407.9
30	5485 3	2202.3	30	5985.6	2556.2	30	6533.8	2960.4	30	7139.4	3424.5
40	5501.3	2213.4	40	6003.1	2568.8	40	6553.0	2974.9	40	7160.7	3441.1
50	5517.3	2224.5	50	6020.6	2581.5	50	6572.3	2989.4	50	7182.2	3457.8

TABLE VII.—TANGENTS AND EXTERNALS FOR A 1° CURVE.

Angle Δ	Tangent *T*	External *E*	Angle Δ	Tangent *T*	External *E*	Angle Δ	Tangent *T*	External *E*	Angle Δ	Tangent *T*	External *E*
103°	7203.6	3474.6	108°	7886.7	4018.5	113°	8657.1	4651.6	118°	9536.3	5395.4
10′	7225.2	3491.5	10′	7910.9	4038.0	10′	8684.5	4674.5	10′	9567.8	5422.4
20	7246.8	3508.4	20	7935.1	4057.7	20	8712.0	4697.5	20	9599.5	5449.6
30	7268.5	3525.5	30	7959.5	4077.5	30	8739.7	4720.6	30	9631.3	5476.9
40	7290.3	3542.6	40	7983.9	4097.3	40	8767.5	4743.9	40	9663.2	5504.4
50	7312.1	3559.8	50	8008.5	4117.3	50	8795.4	4767.2	50	9695.3	5532.0
104	7334.1	3577.0	109	8033.2	4137.4	114	8823.4	4790.7	119	9727.6	5559.8
10	7356.1	3594.4	10	8057.9	4157.5	10	8851.6	4814.4	10	9760.0	5587.8
20	7378.2	3611.9	20	8082.8	4177.8	20	8879.9	4838.1	20	9792.7	5615.9
30	7400.4	3629.4	30	8107.8	4198.2	30	8908.3	4862.0	30	9825.4	5644.2
40	7422.7	3647.0	40	8132.8	4218.6	40	8936.8	4886.0	40	9858.3	5672.6
50	7445.0	3664.8	50	8158.0	4239.3	50	8965.5	4910.2	50	9891.4	5701.3
105	7467.5	3682.6	110	8183.3	4260.0	115	8994.3	4934.4	120	9924.7	5730.0
10	7490.0	3700.4	10	8208.7	4280.8	10	9023.3	4958.9	10	9958.1	5759.0
20	7512.6	3718.4	20	8234.2	4301.7	20	9052.3	4983.4	20	9991.7	5788.1
30	7535.3	3736.5	30	8259.8	4322.7	30	9081.5	5008.1	30	10025.4	5817.4
40	7558.1	3754.6	40	8285.5	4343.8	40	9110.8	5032.9	40	10059.3	5846.8
50	7581.0	3772.9	50	8311.3	4365.1	50	9140.3	5057.9	50	10093.5	5876.5
106	7604.0	3791.2	111	8337.2	4386.4	116	9169.9	5083.0			
10	7627.0	3809.6	10	8363.2	4407.8	10	9199.7	5108.2			
20	7650.2	3828.2	20	8389.4	4429.5	20	9229.6	5133.6			
30	7673.4	3846.8	30	8415.6	4451.2	30	9259.6	5159.1			
40	7696.7	3865.4	40	8442.0	4473.0	40	9289.8	5184.8			
50	7720.2	3884.2	50	8468.5	4494.9	50	9320.1	5210.6			
107	7743.7	3903.1	112	8495.1	4516.9	117	9350.5	5236.6			
10	7767.3	3922.1	10	8521.8	4539.1	10	9381.1	5262.6			
20	7791.0	3941.2	20	8548.6	4561.3	20	9411.9	5288.9			
30	7814.8	3960.4	30	8575.6	4583.8	30	9442.7	5315.3			
40	7838.6	3979.6	40	8602.6	4606.2	40	9473.8	5341.8			
50	7862.6	3999.0	50	8629.8	4628.9	50	9505.0	5368.5			

TABLE VIII. REDUCTION OF THE MERCURIAL BAROMETER TO STANDARD FOR TEMPERATURE.

Att. Ther.	Height of the barometer in inches.												
	19	20	21	22	23	24	25	26	27	28	29	30	31
F.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
0°	+.050	.052	.055	.057	.060	.063	.065	.068	.070	.073	.076	.078	.081
2	.046	.049	.051	.053	.056	.058	.061	.063	.065	.068	.070	.073	.075
4	.043	.045	.047	.049	.052	.054	.056	.058	.061	.063	.065	.067	.070
6	.039	.041	.043	.045	.047	.049	.052	.054	.056	.058	.060	.062	.064
8	.036	.038	.039	.041	.043	.045	.047	.049	.051	.053	.054	.056	.058
10	.032	.034	.036	.037	.039	.041	.042	.044	.046	.047	.049	.051	.053
12	+.029	.030	.032	.033	.035	.036	.038	.039	.041	.042	.044	.045	.047
14	.025	.027	.028	.029	.031	.032	.033	.035	.036	.037	.039	.040	.041
16	.022	.023	.024	.025	.026	.028	.029	.030	.031	.032	.033	.034	.036
18	.018	.019	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029	.030
20	.015	.016	.016	.017	.018	.019	.020	.020	.021	.022	.023	.024	.024
22	+.011	.012	.013	.013	.014	.014	.015	.016	.016	.017	.017	.018	.019
24	.008	.008	.009	.009	.010	.010	.011	.011	.011	.012	.012	.013	.013
26	.005	.005	.005	.005	.005	.006	.006	.006	.006	.007	.007	.007	.007
28	+.001	.001	.001	.001	.001	.001	.001	.001	.002	.002	.002	.002	.002
30	−.002	.002	.003	.003	.003	.003	.003	.003	.003	.003	.004	.004	.004
32	−.006	.006	.006	.007	.007	.007	.008	.008	.008	.009	.009	.009	.009
34	.009	.010	.010	.011	.011	.012	.012	.013	.013	.014	.014	.015	.015
36	.013	.013	.014	.015	.015	.016	.017	.017	.018	.019	.019	.020	.021
38	.016	.017	.018	.019	.020	.020	.021	.022	.023	.024	.025	.026	.026
40	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029	.030	.031	.032
42	−.023	.024	.025	.027	.028	.029	.030	.032	.033	.034	.035	.036	.038
44	.026	.028	.029	.031	.032	.033	.035	.036	.038	.039	.040	.042	.043
46	.030	.031	.033	.035	.036	.038	.039	.041	.043	.044	.046	.047	.049
48	.033	.035	.037	.039	.040	.042	.044	.046	.047	.049	.051	.053	.054
50	.037	.039	.041	.043	.045	.046	.048	.050	.052	.054	.056	.058	.060
52	−.040	.042	.044	.047	.049	.051	.053	.055	.057	.059	.061	.064	.066
54	.044	.046	.048	.051	.053	.055	.057	.060	.062	.064	.067	.069	.071
56	.047	.050	.052	.055	.057	.060	.062	.064	.067	.069	.072	.074	.077
58	.051	.053	.056	.059	.061	.064	.066	.069	.072	.074	.077	.080	.082
60	.054	.057	.060	.062	.065	.068	.071	.074	.077	.080	.082	.085	.088
62	−.057	.060	.063	.066	.069	.073	.076	.079	.082	.085	.088	.091	.094
64	.061	.064	.067	.070	.074	.077	.080	.083	.086	.090	.093	.096	.099
66	.064	.068	.071	.074	.078	.081	.085	.088	.091	.095	.098	.101	.105
68	.068	.071	.075	.078	.082	.085	.089	.093	.096	.100	.103	.107	.110
70	.071	.075	.079	.082	.086	.090	.094	.097	.101	.105	.109	.112	.116
72	−.075	.078	.082	.086	.090	.094	.098	.102	.106	.110	.114	.118	.122
74	.078	.082	.086	.090	.094	.098	.103	.107	.111	.115	.119	.123	.127
76	.081	.086	.090	.094	.098	.103	.107	.111	.116	.120	.124	.128	.133
78	.085	.089	.094	.098	.103	.107	.112	.116	.120	.125	.129	.134	.138
80	.088	.093	.097	.102	.107	.111	.116	.121	.125	.130	.135	.139	.144
82	−.092	.096	.101	.106	.111	.116	.121	.125	.130	.135	.140	.145	.149
84	.095	.100	.105	.110	.115	.120	.125	.130	.135	.140	.145	.150	.155
86	.098	.104	.109	.114	.119	.124	.130	.135	.140	.145	.150	.155	.161
88	.102	.107	.113	.118	.123	.129	.134	.139	.145	.150	.155	.161	.166
90	.105	.111	.116	.122	.127	.133	.138	.144	.150	.155	.161	.166	.172
92	−.109	.114	.120	.126	.132	.137	.143	.149	.154	.160	.166	.172	.177
94	.112	.118	.124	.130	.136	.142	.147	.153	.159	.165	.171	.177	.183
96	.115	.122	.128	.134	.140	.146	.152	.158	.164	.170	.176	.182	.188
98	.119	.125	.131	.138	.144	.150	.156	.163	.169	.175	.181	.188	.194
100	.122	.129	.135	.142	.148	.154	.161	.167	.174	.180	.187	.193	.200

TABLE IX. BAROMETRIC LEVELING.

Values of $62583.6\ (\log 29.90 - \log B)$.

B	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
Inches.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
17.00	15347	15331	15315	15299	15283	15267	15251	15235	15219	15203
17.10	15187	15172	15156	15140	15124	15108	15092	15076	15061	15045
17.20	15029	15013	14997	14982	14966	14950	14934	14919	14903	14887
17.30	14871	14856	14840	14824	14809	14793	14777	14762	14746	14730
17.40	14715	14699	14684	14668	14652	14637	14621	14606	14590	14575
17.50	14559	14544	14528	14512	14497	14481	14466	14451	14435	14420
17.60	14404	14389	14373	14358	14342	14327	14312	14296	14281	14266
17.70	14250	14235	14219	14204	14189	14173	14158	14143	14128	14112
17.80	14097	14082	14067	14051	14036	14021	14006	13990	13975	13960
17.90	13945	13930	13914	13899	13884	13869	13854	13839	13824	13808
18.00	13793	13778	13763	13748	13733	13718	13703	13688	13673	13658
18.10	13643	13628	13613	13598	13583	13568	13553	13538	13523	13508
18.20	13493	13478	13463	13448	13433	13418	13404	13389	13374	13359
18.30	13344	13329	13314	13300	13285	13270	13255	13240	13226	13211
18.40	13196	13181	13166	13152	13137	13122	13107	13093	13078	13063
18.50	13049	13034	13019	13005	12990	12975	12961	12946	12931	12917
18.60	12902	12888	12873	12858	12844	12829	12815	12800	12785	12771
18.70	12756	12742	12727	12713	12698	12684	12669	12655	12640	12626
18.80	12611	12597	12583	12568	12554	12539	12525	12510	12496	12482
18.90	12467	12453	12438	12424	12410	12395	12381	12367	12352	12338
19.00	12324	12310	12295	12281	12267	12252	12238	12224	12210	12195
19.10	12181	12167	12153	12138	12124	12110	12096	12082	12068	12053
19.20	12039	12025	12011	11997	11983	11969	11954	11940	11926	11912
19.30	11898	11884	11870	11856	11842	11828	11814	11800	11786	11772
19.40	11758	11744	11730	11716	11702	11688	11674	11660	11646	11632
19.50	11618	11604	11590	11576	11562	11548	11534	11520	11507	11493
19.60	11479	11465	11451	11437	11423	11410	11396	11382	11368	11354
19.70	11340	11327	11313	11299	11285	11272	11258	11244	11230	11217
19.80	11203	11189	11175	11162	11148	11134	11121	11107	11093	11080
19.90	11066	11052	11039	11025	11011	10998	10984	10970	10957	10943
20.00	10930	10916	10903	10889	10875	10862	10848	10835	10821	10808
20.10	10794	10781	10767	10754	10740	10727	10713	10700	10686	10673
20.20	10659	10646	10632	10619	10605	10592	10579	10565	10552	10538
20.30	10525	10512	10498	10485	10472	10458	10445	10431	10418	10405
20.40	10391	10378	10365	10352	10338	10325	10312	10298	10285	10272
20.50	10259	10245	10232	10219	10206	10192	10179	10166	10153	10139
20.60	10126	10113	10100	10087	10074	10060	10047	10034	10021	10008
20.70	9995	9982	9968	9955	9942	9929	9916	9903	9890	9877
20.80	9864	9851	9838	9825	9812	9799	9786	9772	9759	9746
20.90	9733	9720	9707	9694	9681	9668	9655	9642	9629	9617
21.00	9604	9591	9578	9565	9552	9539	9526	9513	9500	9487
21.10	9474	9462	9449	9436	9423	9410	9397	9384	9372	9359
21.20	9346	9333	9320	9307	9295	9282	9269	9256	9244	9231
21.30	9218	9205	9193	9180	9167	9154	9142	9129	9116	9103
21.40	9091	9078	9065	9053	9040	9027	9015	9002	8989	8977
21.50	8964	8951	8939	8926	8913	8901	8888	8876	8863	8850
21.60	8838	8825	8813	8800	8788	8775	8762	8750	8737	8725
21.70	8712	8700	8687	8675	8662	8650	8637	8625	8612	8600
21.80	8587	8575	8562	8550	8538	8525	8513	8500	8488	8475
21.90	8463	8451	8438	8426	8413	8401	8389	8376	8364	8352
22.00	8339	8327	8314	8302	8290	8277	8265	8253	8240	8228

TABLE IX. BAROMETRIC LEVELING (Continued).

Values of 62583.6 (log 29.90 − log B).

B	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
Inches.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet
22.00	8339	8327	8314	8302	8290	8277	8265	8253	8240	8228
22.10	8216	8204	8191	8179	8167	8154	8142	8130	8118	8105
22.20	8093	8081	8069	8056	8044	8032	8020	8008	7995	7983
22.30	7971	7959	7947	7935	7922	7910	7898	7886	7874	7862
22.40	7849	7837	7825	7813	7801	7789	7777	7765	7753	7740
22.50	7728	7716	7704	7692	7680	7668	7656	7644	7632	7620
22.60	7608	7596	7584	7572	7560	7548	7536	7524	7512	7500
22.70	7488	7476	7464	7452	7440	7428	7416	7404	7392	7380
22.80	7368	7356	7345	7333	7321	7309	7297	7285	7273	7261
22.90	7249	7238	7226	7214	7202	7190	7178	7166	7155	7143
23.00	7131	7119	7107	7096	7084	7072	7060	7048	7037	7025
23.10	7013	7001	6990	6978	6966	6954	6943	6931	6919	6907
23.20	6896	6884	6872	6861	6849	6837	6825	6814	6802	6790
23.30	6779	6767	6755	6744	6732	6721	6709	6697	6686	6674
23.40	6662	6651	6639	6628	6616	6604	6593	6581	6570	6558
23.50	6546	6535	6523	6512	6500	6489	6477	6466	6454	6443
23.60	6431	6420	6408	6397	6385	6374	6362	6351	6339	6328
23.70	6316	6305	6293	6282	6270	6259	6247	6236	6225	6213
23.80	6202	6190	6179	6167	6156	6145	6133	6122	6110	6099
23.90	6088	6076	6065	6054	6042	6031	6020	6008	5997	5986
24.00	5974	5963	5952	5940	5929	5918	5906	5895	5884	5872
24.10	5861	5850	5839	5827	5816	5805	5794	5782	5771	5760
24.20	5749	5737	5726	5715	5704	5693	5681	5670	5659	5648
24.30	5637	5625	5614	5603	5592	5581	5570	5558	5547	5536
24.40	5525	5514	5503	5492	5480	5469	5458	5447	5436	5425
24.50	5414	5403	5392	5381	5369	5358	5347	5336	5325	5314
24.60	5303	5292	5281	5270	5259	5248	5237	5226	5215	5204
24.70	5193	5182	5171	5160	5149	5138	5127	5116	5105	5094
24.80	5083	5072	5061	5050	5039	5028	5017	5006	4995	4985
24.90	4974	4963	4952	4941	4930	4919	4908	4897	4886	4876
25.00	4865	4854	4843	4832	4821	4810	4800	4789	4778	4767
25.10	4756	4745	4735	4724	4713	4702	4691	4681	4670	4659
25.20	4648	4637	4627	4616	4605	4594	4584	4573	4562	4551
25.30	4540	4530	4519	4508	4498	4487	4476	4465	4455	4444
25.40	4433	4423	4412	4401	4391	4380	4369	4358	4348	4337
25.50	4326	4316	4305	4295	4284	4273	4263	4252	4241	4231
25.60	4220	4209	4199	4188	4178	4167	4156	4146	4135	4125
25.70	4114	4104	4093	4082	4072	4061	4051	4040	4030	4019
25.80	4009	3998	3988	3977	3966	3956	3945	3935	3924	3914
25.90	3903	3893	3882	3872	3861	3851	3841	3830	3820	3809
26.00	3799	3788	3778	3767	3757	3746	3736	3726	3715	3705
26.10	3694	3684	3674	3663	3653	3642	3632	3622	3611	3601
26.20	3590	3580	3570	3559	3549	3539	3528	3518	3508	3497
26.30	3487	3477	3466	3456	3446	3435	3425	3415	3404	3394
26.40	3384	3373	3363	3353	3343	3332	3322	3312	3301	3291
26.50	3281	3270	3260	3250	3240	3230	3219	3209	3199	3189

TABLE IX. BAROMETRIC LEVELING (Continued).

Values of 62583.6 (log 29.90 − log B).

B	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
Inches.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
26.50	3281	3270	3260	3250	3240	3230	3219	3209	3199	3189
26.60	3179	3168	3158	3148	3138	3128	3117	3107	3097	3087
26.70	3077	3066	3056	3046	3036	3026	3016	3005	2995	2985
26.80	2975	2965	2955	2945	2934	2924	2914	2904	2894	2884
26.90	2874	2864	2854	2843	2833	2823	2813	2803	2793	2783
27.00	2773	2763	2753	2743	2733	2723	2713	2703	2692	2682
27.10	2672	2662	2652	2642	2632	2622	2612	2602	2592	2582
27.20	2572	2562	2552	2542	2532	2522	2512	2502	2493	2483
27.30	2473	2463	2453	2443	2433	2423	2413	2403	2393	2383
27.40	2373	2363	2353	2343	2334	2324	2314	2304	2294	2284
27.50	2274	2264	2254	2245	2235	2225	2215	2205	2195	2185
27.60	2176	2166	2156	2146	2136	2126	2116	2107	2097	2087
27.70	2077	2067	2058	2048	2038	2028	2018	2009	1999	1989
27.80	1979	1970	1960	1950	1940	1930	1921	1911	1901	1891
27.90	1882	1872	1862	1852	1843	1833	1823	1814	1804	1794
28.00	1784	1775	1765	1755	1746	1736	1726	1717	1707	1697
28.10	1688	1678	1668	1859	1649	1639	1630	1620	1610	1601
28.20	1591	1581	1572	1562	1552	1543	1533	1524	1514	1503
28.30	1495	1485	1476	1466	1456	1447	1437	1428	1418	1404
28.40	1399	1389	1380	1370	1361	1351	1342	1332	1322	1318
28.50	1303	1294	1284	1275	1265	1256	1246	1237	1227	1218
28.60	1208	1199	1189	1180	1170	1161	1151	1142	1132	1123
28.70	1113	1104	1094	1085	1075	1066	1057	1047	1038	1028
28.80	1019	1009	1000	990	981	972	962	953	943	934
28.90	925	915	906	896	887	878	868	859	849	840
29.00	831	821	812	803	793	784	775	765	756	746
29.10	737	728	718	709	700	690	681	672	663	653
29.20	644	635	625	616	607	597	588	579	570	560
29.30	551	542	532	523	514	505	495	486	477	468
29.40	458	449	440	431	421	412	403	394	384	375
29.50	366	357	348	338	329	320	311	302	292	283
29.60	274	265	256	247	237	228	219	210	201	192
29.70	182	173	164	155	146	137	128	118	109	100
29.80	+ 91	+ 82	+ 73	+ 64	+ 55	+ 45	+ 36	+ 27	+ 18	+ 9
29.90	0	− 9	− 18	− 27	− 36	− 45	− 55	− 64	− 73	− 82
30.00	− 91	−100	−109	− 118	− 127	− 136	− 145	− 154	− 163	− 172
30.10	−181	−190	−199	− 208	− 217	− 226	− 235	− 244	− 253	− 262
30.20	−271	−280	−289	− 298	− 307	− 316	− 325	− 334	− 343	− 352
30.30	−361	−370	−379	− 388	− 397	− 406	− 415	− 424	− 433	− 442
30.40	−451	−460	−469	− 478	− 486	− 495	− 504	− 513	− 522	− 531
30.50	−540	−549	−558	− 567	− 576	− 585	− 593	− 602	− 611	− 620
30.60	−629	−638	−647	− 656	− 665	− 673	− 682	− 691	− 700	− 709
30.70	−718	−727	−735	− 744	− 753	− 762	− 771	− 780	− 788	− 797
30.80	−806	−815	−824	− 833	− 841	− 850	− 859	− 868	− 877	− 885
30.90	−894	−903	−912	− 921	− 929	− 938	− 947	− 956	− 964	− 973
31.00	−982	−991	−999	−1008	−1017	−1026	−1035	−1043	−1052	−1061

TABLE X. BAROMETRIC LEVELING.

Correction for Temperature: + above 50° F. − below 50° F.

Mean t.		Approximate Difference of Height from Table IX.												
		20	40	60	80	100	200	300	400	500	600	700	800	900
F.	F.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.
49°	51°							1	1	1	1	1	2	2
48	52						1	1	2	2	2	3	3	4
47	53					1	1	2	2	3	4	4	5	6
46	54				1	1	2	2	3	4	5	6	7	7
45	55			1	1	1	2	3	4	5	6	7	8	9
44	56			1	1	1	2	4	5	6	7	9	10	11
43	57		1	1	1	1	3	4	6	7	9	10	11	13
42	58		1	1	1	2	3	5	7	8	10	11	13	15
41	59		1	1	1	2	4	6	7	9	11	13	15	17
40	60		1	1	2	2	4	6	8	10	12	14	16	18
39	61		1	1	2	2	4	7	9	11	13	16	18	20
38	62		1	1	2	2	5	7	10	12	15	17	20	22
37	63	1	1	2	2	3	5	8	11	13	16	19	21	24
36	64	1	1	2	2	3	6	9	11	14	17	20	23	26
35	65	1	1	2	2	3	6	9	12	15	18	21	24	28
34	66	1	1	2	3	3	7	10	13	16	20	23	26	29
33	67	1	1	2	3	3	7	10	14	17	21	24	28	31
32	68	1	1	2	3	4	7	11	15	18	22	26	29	33
31	69	1	2	2	3	4	8	12	15	19	23	27	31	35
30	70	1	2	2	3	4	8	12	16	20	24	29	33	37
29	71	1	2	3	3	4	9	13	17	21	26	30	34	39
28	72	1	2	3	4	4	9	13	18	22	27	31	36	40
27	73	1	2	3	4	5	9	14	19	23	28	33	38	42
26	74	1	2	3	4	5	10	15	20	24	29	34	39	44
25	75	1	2	3	4	5	10	15	20	25	31	36	41	46
24	76	1	2	3	4	5	11	16	21	27	32	37	42	48
23	77	1	2	3	4	6	11	17	22	28	33	39	44	50
22	78	1	2	3	5	6	11	17	23	29	34	40	46	51
21	79	1	2	4	5	6	12	18	24	30	35	41	47	53
20	80	1	2	4	5	6	12	18	24	31	37	43	49	55
19	81	1	3	4	5	6	13	19	25	32	38	44	51	57
18	82	1	3	4	5	7	13	20	26	33	39	46	52	59
17	83	1	3	4	5	7	13	20	27	34	40	47	54	61
16	84	1	3	4	6	7	14	21	28	35	42	49	55	62
15	85	1	3	4	6	7	14	21	29	36	43	50	57	64
14	86	1	3	4	6	7	15	22	29	37	44	51	59	66
13	87	2	3	5	6	8	15	23	30	38	45	53	60	68
12	88	2	3	5	6	8	15	23	31	39	46	54	62	70
11	89	2	3	5	6	8	16	24	32	40	48	56	64	72
10	90	2	3	5	7	8	16	24	33	41	49	57	65	73
9	91	2	3	5	7	8	17	25	33	42	50	59	67	75
8	92	2	3	5	7	9	17	26	34	43	51	60	69	77
7	93	2	4	5	7	9	18	26	35	44	53	61	70	79
6	94	2	4	5	7	9	18	27	36	45	54	63	72	81
5	95	2	4	6	7	9	18	28	37	46	55	64	73	83
4	96	2	4	6	8	9	19	28	38	47	56	66	75	84
3	97	2	4	6	8	10	19	29	38	48	57	67	77	86
2	98	2	4	6	8	10	20	29	39	49	59	69	78	88
1	99	2	4	6	8	10	20	30	40	50	60	70	80	90
0	100	2	4	6	8	10	20	31	41	51	61	71	82	92

TABLE XI. POINT SWITCH TURNOUTS FROM STRAIGHT TRACK.

Gage 4 feet 8½ inches. Spread at heel 5½ inches.

Frog No., *n*.	Frog Angle, *F*.	Length of Switch, *l*.	Switch Angle, *S*.	Radius of Turnout Curve, *r*.	Degree of Turnout Curve.	Point Lead, *BF*.	Middle Ordinate, Outside Rail.	Quarter Point Ord.	Crotch Frog Angle, *F″*.	Crotch Frog No., *n″*.	Point Lead to Crotch Frog.	Frog No., *n*.
	° ′		° ′	Feet.	° ′	Feet.	Ins.	Ins.	° ′			
6	9 31½	15	1 45	316.56	18 11	58.04	8 13/16	6⅝	12 59	4.39	41.31	6
6½	8 48	15	"	373.81	15.22	61.04	8½	6⅜	12 02	4.75	42.93	6½
7	8 10	15	"	436.47	13 09	63.96	8¼	6 3/16	11 13	5.09	44.48	7
7½	7 37½	15	"	504.67	11 22	66.81	8	6	10 30½	5.44	45.98	7½
8	7 09	15	"	578.60	9 55	69.58	7¾	5 13/16	9 54½	5.77	47.37	8
8½	6 44	15	"	658.60	8 42	72.30	7½	5⅝	9 22	6.11	48.74	8½
9	6 21½	15	"	744.97	7 42	74.94	7¼	5 7/16	8 53½	6.43	50.01	9
9½	6 01½	15	"	837.94	6 51	77.53	7	5¼	8 27½	6.76	51.30	9½
10	5 43½	15	"	937.82	6 07	80.05	6¾	5⅛	8 04	7.09	52.52	10
8	7 09	18	1 27½	567.50	10 07	74.44	8 7/16	6⅜	9 48	5.83	52.12	8
8½	6 44	18	"	644.27	8 54	77.35	8⅛	6⅛	9 15½	6.18	53.66	8½
9	6 21½	18	"	726.70	7 53	80.19	8	6	8 46½	6.52	55.12	9
9½	6 01½	18	"	814.91	7 02	82.98	7¾	5⅞	8 20½	6.86	56.56	9½
10	5 43½	18	"	909.09	6 18	85.71	7⅝	5 1/16	7 56	7.21	58.00	10
10½	5 27	18	"	1009.50	5 41	88.38	7⅜	5½	7 36	7.53	59.29	10½
11	5 12	18	"	1116.33	5 08	90.99	7⅛	5⅜	7 17	7.85	60.55	11
11½	4 59	18	"	1229.72	4 40	93.56	7	5¼	6 59	8.19	61.84	11½
12	4 46½	18	"	1350.35	4 15	96.08	6¾	5⅛	6 43½	8.51	63.01	12
10	5 43½	20	1 19	897.12	6 23	89.11	8	6	7 53	7.26	61.33	10
10½	5 27	20	"	994.78	5 46	91.90	7 13/16	5⅞	7 32½	7.59	62.71	10½
11	5 12	20	"	1098.38	5 13	94.63	7⅝	5¾	7 13½	7.92	64.06	11
11½	4 59	20	"	1207.98	4 45	97.31	7 7/16	5 9/16	6 56	8.26	65.44	11½
12	4 46½	20	"	1324.18	4 20	99.96	7¼	5 7/16	6 40	8.58	66.71	12
				Spread at Heel 5 inches.								
4	14 15	12	1 59½	139.92	41 53	42.08	9¾	7 5/16	19 21	2.93	30.97	4
4½	12 41	12	"	178.07	32 37	45.34	9 7/16	7 1/16	17 16½	3.29	32.83	4½
5	11 25	15	1 35½	218.69	26 26	52.63	9¾	7 5/16	15 31	3.67	38.69	5
5½	10 23½	15	"	265.75	21 41	55.90	9½	7⅛	14 09½	4.03	40.56	5½
6	9 31½	15	"	317.72	18 07	59.09	9⅛	6⅞	13 00½	4.38	42.37	6
6½	8 48	15	"	374.80	15 20	62.21	8 15/16	6 11/16	12 02½	4.74	44.12	6½
7	8 10	15	"	437.11	13 08	65.25	8 11/16	6½	11 13½	5.09	45.78	7
7½	7 37½	15	"	504.75	11 22	68.23	8 7/16	6 5/16	10 30½	5.44	47.39	7½
8	7 09	15	"	577.88	9 56	71.13	8 3/16	6⅛	9 54	5.77	48.90	8
8½	6 44	15	"	656.80	8 44	73.97	8	6	9 21	6.11	50.39	8½
9	6 21½	15	"	741.72	7 44	76.75	7¾	5¾	8 53	6.44	51.77	9
9½	6 01½	15	"	832.82	6 53	79.47	7½	5⅝	8 26	6.78	53.19	9½
10	5 43½	15	"	930.35	6 10	82.13	7¼	5½	8 02	7.12	54.56	10

From Camp's Notes on Track, by permission.

TABLE XII. MIDDLE ORDINATES FOR CURVING RAILS IN INCHES.

D	Length of Rail Chord in Feet												D
	10	15	20	24	26	27	28	29	30	31	32	33	
°													°
2		1/8	1/4	1/4	3/8	3/8	3/8	1/2	1/2	1/2	1/2	5/8	2
3	1/8	1/8	3/8	1/2	1/2	5/8	5/8	5/8	3/4	3/4	3/4	7/8	3
4	1/8	1/4	3/8	5/8	3/4	3/4	7/8	7/8	1	1	1 1/8	1 1/8	4
5	1/8	1/4	1/2	3/4	7/8	1	1	1 1/8	1 1/8	1 1/4	1 3/8	1 3/8	5
6	1/8	3/8	5/8	7/8	1	1 1/8	1 1/4	1 3/8	1 3/8	1 1/2	1 5/8	1 3/4	6
7	1/8	3/8	3/4	1	1 1/4	1 3/8	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	7
8	1/4	1/2	7/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4	8
9	1/4	1/2	1	1 3/8	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4	2 3/8	2 5/8	9
10	1/4	5/8	1	1 1/2	1 3/4	1 7/8	2	2 1/4	2 3/8	2 1/2	2 5/8	2 7/8	10
12	3/8	3/4	1 1/4	1 3/4	2 1/8	2 1/4	2 1/2	2 5/8	2 7/8	3	3 1/4	3 3/8	12
14	3/8	7/8	1 1/2	2	2 1/2	2 5/8	2 7/8	3 1/8	3 1/4	3 1/2	3 3/4	4	14
16	3/8	1	1 5/8	2 3/8	2 7/8	3	3 1/4	3 1/2	3 3/4	4	4 1/4	4 1/2	16
18	1/2	1	1 7/8	2 3/4	3 1/8	3 3/8	3 3/4	4	4 1/4	4 1/2	4 7/8	5 1/8	18
20	1/2	1 1/8	2	3	3 1/2	3 7/8	4 1/8	4 3/8	4 3/4	5	5 3/8	5 3/4	20
25	5/8	1 1/2	2 5/8	3 3/4	4 3/8	4 3/4	5 1/8	5 1/2	5 7/8	6 1/4	6 3/4	7 1/8	25
30	3/4	1 3/4	3 1/8	4 1/2	5 1/4	5 3/4	6 1/8	6 5/8	7 1/8	7 1/2	8	8 1/2	30
35	7/8	2	3 5/8	5 1/4	6 1/4	6 5/8	7 1/8	7 3/4	8 1/4	8 3/4	9 3/8	10	35
40	1	2 3/8	4 1/8	6	7 1/8	7 5/8	8 1/4	8 3/4	9 3/8	10	10 3/4	11 3/8	40
45	1 1/8	2 5/8	4 3/4	6 3/4	8	8 5/8	9 1/4	9 7/8	10 5/8	11 1/4	12 1/8	12 7/8	45
50	1 1/4	3	5 1/4	7 1/2	8 7/8	9 1/2	10 1/4	11	11 3/4	12 5/8	13 3/8	14 1/4	50

TABLE XIII. ELEVATION OF OUTER RAIL IN INCHES.

D.	Velocity in Miles per Hour.													D
	10	15	20	25	30	35	40	45	50	55	60	65	70	
°														°
1		1/8	1/4	3/8	5/8	3/4	1 1/8	1 3/8	1 5/8	2	2 3/8	2 3/4	3 1/4	1
2	1/8	3/8	1/2	7/8	1 1/8	1 5/8	2 1/8	2 5/8	3 1/4	4	4 3/4	5 1/2	6 1/2	2
3	1/4	1/2	3/4	1 1/4	1 3/4	2 3/8	3 1/8	4	4 7/8	6	7 1/8	8 3/8	9 3/4	3
4	1/4	5/8	1	1 5/8	2 3/8	3 1/4	4 1/4	5 3/8	6 5/8	8	9 1/2			4
5	3/8	3/4	1 1/4	2	3	4	5 1/4	6 5/8	8 1/4					5
6	3/8	1	1 5/8	2 1/2	3 1/2	4 7/8	6 1/4	8						6
7	1/2	1 1/8	1 7/8	2 7/8	4 1/8	5 5/8	7 3/8							7
8	1/2	1 1/4	2 1/8	3 1/4	4 3/4	6 1/2	8 3/8							8
9	5/8	1 3/8	2 3/8	3 3/4	5 3/8	7 1/4								9
10	3/4	1 1/2	2 5/8	4 1/8	5 7/8	8 1/8								10
11	3/4	1 3/4	2 7/8	4 1/2	6 1/2	8 7/8								11
12	7/8	1 7/8	3 1/8	4 7/8	7 1/8									12
13	7/8	2	3 3/8	5 3/8	7 3/4									13
14	1	2 1/8	3 5/8	5 3/4	8 3/8									14
15	1	2 1/4	3 7/8	6 1/4	8 7/8									15
16	1 1/8	2 1/2	4 1/4	6 5/8										16
17	1 1/4	2 5/8	4 1/2	7										17
18	1 1/4	2 3/4	4 3/4	7 1/2										18
19	1 3/8	2 7/8	5	7 3/4										19
20	1 3/8	3	5 1/4	8 1/8										20

TABLE XIV. CORRECTIONS TO HORIZONTAL DISTANCES FOR STADIA WORK.

In feet per 100 ft.

Vertical Angle.	0′	10′	20′	30′	40′	50′	Vertical Angle.
0°				.01	.01	.02	0°
1	.03	.04	.05	.07	.08	.10	1
2	.12	.14	.17	.19	.22	.24	2
3	.27	.31	.34	.37	.41	.45	3
4	.49	.53	.57	.62	.66	.71	4
5	.76	.81	.86	.92	.98	1.03	5
6	1.09	1.15	1.22	1.28	1.35	1.42	6
7	1.49	1.56	1.63	1.70	1.78	1.86	7
8	1.94	2.02	2.10	2.18	2.27	2.36	8
9	2.45	2.54	2.63	2.72	2.82	2.92	9
10	3.02	3.12	3.22	3.32	3.43	3.53	10
11	3.64	3.75	3.86	3.97	4.09	4.21	11
12	4.32	4.44	4.56	4.68	4.81	4.93	12
13	5.06	5.19	5.32	5.45	5.58	5.72	13
14	5.85	5.99	6.13	6.27	6.41	6.55	14
15	6.70	6.84	6.99	7.14	7.29	7.44	15
16	7.60	7.75	7.91	8.07	8.23	8.39	16
17	8.55	8.71	8.88	9.04	9.21	9.38	17
18	9.55	9.72	9.89	10.07	10.24	10.42	18
19	10.60	10.78	10.96	11.14	11.33	11.51	19
20	11.70	11.89	12.07	12.26	12.46	12.65	20
21	12.84	13.04	13.23	13.43	13.63	13.83	21
22	14.03	14.24	14.44	14.64	14.85	15.06	22
23	15.27	15.48	15.69	15.90	16.11	16.33	23
24	16.54	16.76	16.98	17.20	17.42	17.64	24
25	17.86	18.08	18.31	18.53	18.76	18.99	25
26	19.22	19.45	19.68	19.91	20.14	20.38	26
27	20.61	20.85	21.08	21.32	21.56	21.80	27
28	22.04	22.28	22.52	22.77	23.01	23.26	28
29	23.50	23.75	24.00	24.25	24.50	24.75	29
30	25.00	25.25	25.51	25.76	26.01	26.27	30

TABLE XV. CUBIC YARDS PER 100 FEET. SLOPES $\frac{1}{4}$: 1.

Depth, d.	Base **14**	Base **16**	Base **18**	Base **20**	Base **22**	Base **26**	Base **28**	Base **30**	Base **32**
1	53	60	68	75	82	97	105	112	119
2	107	122	137	152	167	196	211	226	241
3	163	186	208	230	253	297	319	342	364
4	222	252	281	311	341	400	430	459	489
5	282	319	356	394	431	505	542	579	616
6	344	389	433	478	522	611	656	700	745
7	408	460	512	564	616	719	771	823	875
8	474	533	593	652	711	830	889	948	1007
9	542	608	675	742	808	942	1008	1075	1142
10	611	685	759	833	907	1056	1130	1204	1278
11	682	764	845	927	1008	1171	1253	1334	1416
12	756	844	933	1022	1111	1289	1378	1467	1556
13	831	926	1023	1119	1216	1408	1505	1601	1697
14	907	1010	1115	1219	1322	1530	1633	1737	1841
15	986	1096	1208	1320	1431	1653	1764	1875	1986
16	1067	1184	1304	1422	1541	1778	1896	2015	2133
17	1149	1274	1401	1527	1653	1905	2031	2156	2282
18	1233	1366	1500	1633	1767	2033	2167	2300	2433
19	1319	1460	1601	1742	1882	2164	2305	2445	2586
20	1407	1555	1704	1852	2000	2296	2444	2593	2741
21	1497	1653	1808	1964	2119	2431	2586	2742	2897
22	1589	1752	1915	2078	2241	2567	2730	2893	3056
23	1682	1853	2023	2194	2364	2705	2875	3045	3216
24	1778	1955	2133	2311	2489	2844	3022	3200	3378
25	1875	2060	2245	2431	2616	2986	3171	3356	3542
26	1974	2166	2359	2552	2744	3130	3322	3515	3707
27	2075	2274	2475	2675	2875	3275	3475	3675	3875
28	2178	2384	2593	2800	3007	3422	3630	3837	4044
29	2282	2496	2712	2927	3142	3571	3786	4001	4215
30	2389	2610	2833	3056	3278	3722	3944	4167	4389
31	2497	2726	2956	3186	3416	3875	4105	4334	4564
32	2607	2844	3081	3319	3556	4030	4267	4504	4741
33	2719	2964	3208	3453	3697	4186	4431	4675	4919
34	2833	3085	3337	3589	3841	4344	4596	4848	5100
35	2949	3208	3468	3727	3986	4505	4764	5023	5282
36	3067	3333	3600	3867	4133	4667	4933	5200	5467
37	3186	3460	3734	4008	4282	4831	5105	5378	5653
38	3307	3589	3870	4152	4433	4996	5278	5559	5841
39	3431	3719	4008	4297	4586	5164	5453	5742	6031
40	3556	3852	4148	4445	4741	5333	5630	5926	6222
41	3682	3986	4290	4594	4897	5505	5808	6112	6416
42	3811	4122	4433	4744	5056	5678	5989	6300	6611
43	3942	4260	4579	4897	5216	5853	6171	6490	6808
44	4074	4400	4726	5052	5378	6030	6356	6682	7007
45	4208	4541	4875	5208	5542	6208	6542	6875	7208
46	4344	4684	5026	5367	5707	6389	6730	7070	7411
47	4482	4830	5179	5527	5875	6571	6919	7267	7616
48	4622	4978	5333	5689	6044	6756	7111	7467	7822
49	4764	5127	5490	5853	6216	6942	7305	7668	8031
50	4907	5278	5648	6019	6389	7130	7500	7870	8241
51	5053	5430	5808	6186	6564	7319	7697	8075	8453
52	5200	5584	5970	6356	6741	7511	7896	8281	8667
53	5349	5741	6134	6527	6919	7705	8097	8490	8883
54	5500	5900	6300	6700	7100	7900	8300	8700	9100
55	5653	6060	6468	6875	7282	8097	8505	8912	9319
56	5807	6222	6637	7052	7467	8296	8711	9126	9541
57	5964	6386	6808	7231	7653	8497	8919	9342	9764
58	6122	6552	6981	7411	7841	8700	9130	9559	9989
59	6282	6719	7156	7593	8031	8905	9342	9779	10216
60	6444	6889	7333	7778	8222	9111	9556	10000	19445

TABLE XV. CUBIC YARDS PER 100 FEET. SLOPES $\frac{1}{2}$: 1.

Depth, *d*.	Base. **14**	Base **16**	Base **18**	Base **20**	Base **22**	Base **28**	Base **30**	Base **32**
1	54	61	69	76	83	106	113	120
2	111	126	141	156	170	215	230	244
3	172	194	217	239	261	328	350	372
4	237	267	296	326	356	444	474	504
5	306	343	380	417	454	565	602	639
6	378	422	467	511	556	689	733	778
7	454	506	557	609	661	817	869	920
8	533	593	652	711	770	948	1007	1067
9	617	683	750	817	883	1083	1150	1217
10	704	778	852	926	1000	1222	1297	1370
11	794	876	957	1039	1120	1365	1447	1528
12	889	978	1067	1156	1244	1511	1600	1689
13	987	1083	1180	1276	1372	1661	1757	1854
14	1089	1193	1296	1400	1504	1815	1919	2022
15	1194	1306	1417	1528	1639	1972	2083	2194
16	1304	1422	1541	1659	1779	2133	2252	2370
17	1417	1543	1669	1794	1921	2298	2424	2550
18	1533	1667	1800	1933	2067	2467	2600	2733
19	1654	1794	1935	2076	2217	2639	2780	2920
20	1778	1926	2074	2222	2370	2815	2963	3111
21	1906	2061	2217	2372	2528	2994	3150	3305
22	2037	2200	2363	2526	2689	3178	3341	3504
23	2172	2343	2513	2683	2854	3365	3535	3706
24	2311	2489	2667	2844	3022	3556	3733	3911
25	2454	2639	2824	3009	3194	3750	3935	4120
26	2600	2793	2985	3178	3370	3948	4141	4332
27	2750	2950	3150	3350	3550	4151	4350	4550
28	2904	3111	3319	3526	3733	4356	4563	4771
29	3061	3276	3491	3706	3920	4565	4780	4994
30	3222	3444	3667	3889	4111	4778	5000	5222
31	3387	3617	3846	4076	4306	4994	5224	5454
32	3556	3793	4030	4267	4504	5215	5452	5689
33	3728	3972	4217	4461	4706	5439	5683	5928
34	3904	4156	4407	4659	4911	5667	5919	6170
35	4083	4343	4602	4861	5120	5898	6157	6417
36	4267	4533	4800	5067	5333	6133	6400	6667
37	4454	4728	5002	5276	5550	6372	6646	6920
38	4644	4926	5207	5489	5770	6615	6896	7178
39	4839	5128	5417	5706	5994	6861	7150	7439
40	5037	5333	5630	5926	6222	7111	7408	7704
41	5239	5543	5846	6150	6454	7365	7669	7972
42	5444	5756	6067	6378	6689	7622	7933	8244
43	5654	5972	6291	6609	6928	7883	8202	8520
44	5867	6193	6519	6844	7170	8148	8474	8800
45	6083	6417	6750	7083	7417	8417	8750	9083
46	6304	6644	6985	7326	7667	8689	9030	9370
47	6528	6876	7224	7572	7920	8965	9313	9661
48	6756	7111	7467	7822	8178	9244	9600	9956
49	6987	7350	7713	8076	8439	9528	9891	10254
50	7222	7593	7963	8333	8704	9815	10185	10556
51	7461	7839	8217	8594	8972	10106	10483	10861
52	7704	8089	8474	8859	9244	10400	10785	11170
53	7950	8343	8735	9128	9520	10698	11091	11483
54	8200	8600	9000	9400	9800	11000	11400	11800
55	8454	8861	9269	9676	10083	11306	11713	12121
56	8711	9126	9541	9956	10370	11615	12030	12445
57	8972	9394	9817	10239	10661	11928	12350	12772
58	9237	9667	10096	10526	10956	12244	12674	13104
59	9506	9943	10380	10817	11254	12565	13002	13439
60	9778	10222	10667	11111	11556	12889	13333	13778

TABLE XV. CUBIC YARDS PER 100 FEET. SLOPE 1:1.

Depth, *d.*	Base **14**	Base **16**	Base **18**	Base **20**	Base **22**	Base **28**	Base **30**	Base **32**
1	56	63	70	78	85	107	115	122
2	119	133	148	163	178	222	237	252
3	189	211	233	256	278	344	367	389
4	267	296	326	356	385	474	504	533
5	352	389	426	463	500	611	648	685
6	444	489	533	578	622	756	800	844
7	544	596	648	700	752	907	959	1011
8	652	711	770	830	889	1067	1126	1185
9	767	833	900	967	1033	1233	1300	1367
10	889	963	1037	1111	1185	1407	1481	1556
11	1019	1100	1181	1263	1344	1589	1670	1752
12	1156	1244	1333	1422	1511	1778	1867	1956
13	1300	1396	1493	1589	1685	1974	2070	2167
14	1452	1556	1659	1763	1867	2178	2281	2385
15	1611	1722	1833	1944	2056	2389	2500	2611
16	1778	1896	2015	2133	2252	2607	2726	2844
17	1952	2078	2204	2330	2456	2833	2959	3085
18	2133	2267	2400	2533	2667	3067	3200	3333
19	2322	2463	2604	2744	2885	3307	3448	3589
20	2519	2667	2815	2963	3111	3556	3704	3852
21	2722	2878	3033	3189	3344	3811	3967	4122
22	2933	3096	3259	3422	3585	4074	4237	4400
23	3152	3322	3493	3663	3833	4344	4515	4685
24	3378	3556	3733	3911	4089	4622	4800	4978
25	3611	3796	3981	4167	4352	4907	5093	5278
26	3852	4044	4237	4430	4622	5200	5393	5585
27	4100	4300	4500	4700	4900	5500	5700	5900
28	4356	4563	4770	4978	5185	5807	6015	6222
29	4619	4833	5048	5263	5478	6122	6337	6552
30	4889	5111	5333	5556	5778	6444	6667	6889
31	5167	5396	5626	5856	6085	6774	7004	7233
32	5452	5689	5926	6163	6399	7111	7348	7585
33	5744	5989	6233	6478	6722	7456	7700	7944
34	6044	6296	6548	6800	7052	7807	8059	8311
35	6352	6611	6870	7130	7389	8167	8426	8685
36	6667	6933	7200	7467	7733	8533	8800	9067
37	6989	7263	7537	7811	8085	8907	9181	9456
38	7319	7600	7881	8163	8444	9289	9570	9852
39	7656	7944	8233	8522	8811	9678	9967	10256
40	8000	8296	8593	8889	9185	10074	10370	10667
41	8352	8656	8959	9263	9567	10478	10781	11085
42	8711	9022	9333	9644	9956	10889	11200	11511
43	9078	9396	9715	10033	10352	11307	11626	11944
44	9452	9778	10104	10430	10756	11733	12059	12385
45	9833	10167	10500	10833	11167	12167	12500	12833
46	10222	10563	10904	11244	11585	12607	12948	13289
47	10619	10967	11315	11663	12011	13056	13404	13752
48	11022	11378	11733	12089	12444	13511	13867	14222
49	11433	11796	12159	12522	12885	13974	14337	14700
50	11852	12222	12593	12963	13333	14444	14815	15185
51	12278	12656	13033	13411	13790	14922	15300	15678
52	12711	13096	13481	13867	14252	15407	15793	16178
53	13152	13544	13937	14330	14722	15900	16293	16685
54	13600	14000	14400	14800	15200	16400	16800	17200
55	14056	14463	14870	15278	15685	16907	17315	17722
56	14519	14933	15348	15763	16178	17422	17837	18252
57	14989	15411	15833	16256	16678	17944	18367	18789
58	15467	15896	16326	16756	17185	18474	18904	19333
59	15952	16389	16826	17263	17700	19011	19448	19885
60	16444	16889	17333	17778	18222	19556	20000	20244

TABLE XV. CUBIC YARDS PER 100 FEET. SLOPES 1½ : 1.

Depth, *d*.	Base 14	Base 16	Base 18	Base 20	Base 22	Base 28	Base 30	Base 32
1	57	65	72	80	87	109	117	124
2	126	141	156	170	185	230	244	259
3	206	228	250	272	294	361	383	406
4	296	326	356	385	415	504	533	563
5	398	435	472	509	546	657	694	731
6	511	556	600	644	689	822	867	911
7	635	687	739	791	843	998	1050	1102
8	770	830	889	948	1007	1185	1244	1304
9	917	983	1050	1116	1183	1383	1450	1517
10	1074	1148	1222	1296	1370	1593	1667	1741
11	1243	1324	1406	1487	1569	1813	1894	1976
12	1422	1511	1600	1689	1778	2044	2133	2222
13	1613	1709	1806	1902	1998	2287	2383	2480
14	1815	1919	2022	2126	2230	2541	2644	2748
15	2028	2139	2250	2361	2472	2806	2917	3028
16	2252	2370	2489	2607	2726	3081	3200	3319
17	2487	2613	2739	2865	2991	3369	3494	3620
18	2733	2867	3000	3133	3267	3667	3800	3933
19	2991	3131	3272	3413	3554	3976	4117	4257
20	3259	3407	3556	3704	3852	4296	4444	4592
21	3539	3694	3850	4005	4161	4628	4783	4939
22	3830	3993	4156	4318	4482	4970	5133	5296
23	4131	4302	4472	4642	4813	5324	5494	5665
24	4444	4622	4800	4978	5156	5689	5867	6044
25	4769	4954	5139	5324	5509	6065	6250	6435
26	5104	5296	5489	5681	5874	6452	6644	6837
27	5450	5650	5850	6050	6250	6850	7050	7250
28	5807	6015	6222	6430	6637	7259	7467	7674
29	6176	6391	6606	6820	7035	7680	7894	8109
30	6556	6778	7000	7222	7445	8111	8333	8555
31	6946	7176	7406	7635	7865	8554	8783	9013
32	7348	7585	7822	8059	8296	9007	9244	9482
33	7761	8006	8250	8494	8739	9472	9717	9962
34	8185	8437	8689	8941	9193	9948	10200	10452
35	8620	8880	9139	9398	9657	10435	10694	10954
36	9067	9333	9600	9867	10134	10933	11200	11467
37	9524	9798	10072	10346	10621	11443	11717	11991
38	9993	10274	10556	10837	11119	11963	12244	12526
39	10472	10761	11050	11339	11628	12494	12783	13072
40	10963	11259	11556	11852	12148	13037	13333	13630
41	11465	11769	12072	12376	12680	13591	13894	14198
42	11978	12289	12600	12911	13223	14156	14467	14778
43	12502	12820	13139	13457	13776	14731	15050	15369
44	13037	13363	13689	14015	14341	15319	15644	15970
45	13583	13917	14250	14583	14917	15917	16250	16583
46	14141	14481	14822	15163	15504	16526	16867	17207
47	14709	15057	15406	15754	16102	17146	17494	17843
48	15289	15644	16000	16356	16711	17778	18133	18489
49	15880	16243	16606	16968	17332	18420	18783	19146
50	16481	16852	17222	17592	17963	19074	19444	19815
51	17094	17472	17850	18228	18605	19739	20117	20494
52	17719	18104	18489	18874	19259	20415	20800	21185
53	18354	18746	19139	19531	19924	21102	21494	21887
54	19000	19400	19800	20200	20600	21800	22200	22600
55	19657	20065	20472	20880	21287	22509	22917	23324
56	20326	20741	21156	21570	21986	23230	23644	24059
57	21006	21428	21850	22272	22695	23961	24383	24805
58	21696	22126	22556	22985	23415	24704	25133	25563
59	22398	22835	23272	23709	24147	25457	25894	26332
60	23111	23556	24000	24444	24889	26222	26667	27111

TABLE XV. CUBIC YARDS PER 100 FEET. SLOPES 2 : 1.

Depth, d.	Base **14**	Base **16**	Base **18**	Base **20**	Base **22**	Base **28**	Base **30**	Base **32**
1	59	67	74	81	89	111	119	126
2	133	148	163	178	193	237	252	267
3	222	244	267	289	311	378	400	422
4	326	356	385	415	445	533	563	593
5	444	481	519	556	593	704	741	778
6	578	622	667	711	756	889	933	978
7	726	778	830	881	933	1089	1141	1193
8	889	948	1007	1067	1126	1304	1363	1422
9	1067	1133	1200	1267	1333	1533	1600	1667
10	1259	1333	1407	1481	1556	1778	1852	1926
11	1467	1548	1630	1711	1793	2037	2119	2200
12	1689	1778	1867	1956	2044	2311	2400	2489
13	1926	2022	2119	2215	2311	2600	2696	2793
14	2178	2281	2385	2489	2593	2904	3007	3111
15	2444	2556	2667	2778	2889	3222	3333	3444
16	2726	2844	2963	3081	3200	3556	3674	3793
17	3022	3148	3274	3400	3526	3904	4030	4156
18	3333	3467	3600	3733	3867	4267	4400	4533
19	3659	3800	3941	4081	4222	4644	4785	4926
20	4000	4148	4296	4444	4593	5037	5185	5333
21	4356	4511	4667	4822	4978	5444	5600	5756
22	4726	4889	5052	5215	5378	5867	6030	6193
23	5111	5281	5452	5622	5793	6304	6474	6644
24	5511	5689	5867	6044	6222	6756	6933	7111
25	5926	6111	6296	6481	6667	7222	7407	7593
26	6356	6548	6741	6933	7126	7704	7896	8089
27	6800	7000	7200	7400	7600	8200	8400	8600
28	7259	7467	7674	7881	8089	8711	8919	9126
29	7733	7948	8163	8378	8593	9237	9452	9667
30	8222	8444	8667	8889	9111	9778	10000	10222
31	8726	8956	9185	9415	9645	10333	10563	10793
32	9244	9482	9719	9956	10193	10904	11141	11378
33	9778	10022	10267	10511	10756	11489	11733	11978
34	10326	10578	10830	11081	11333	12089	12341	12593
35	10889	11148	11407	11667	11926	12704	12963	13222
36	11467	11733	12000	12267	12533	13333	13600	13867
37	12059	12333	12607	12881	13155	13978	14252	14526
38	12667	12948	13230	13511	13792	14637	14919	15200
39	13289	13578	13867	14156	14444	15311	15600	15889
40	13926	14222	14519	14815	15111	16000	16296	16593
41	14578	14881	15185	15489	15793	16704	17007	17311
42	15244	15556	15867	16178	16489	17422	17733	18044
43	15926	16244	16563	16881	17200	18156	18474	18793
44	16622	16948	17274	17600	17926	18904	19230	19556
45	17333	17667	18000	18333	18667	19667	20000	20333
46	18059	18400	18741	19081	19422	20444	20785	21126
47	18800	19148	19496	19844	20193	21237	21585	21933
48	19556	19911	20267	20622	20978	22044	22400	22756
49	20326	20689	21052	21415	21778	22867	23230	23593
50	21111	21481	21852	22222	22593	23704	24074	24444
51	21911	22289	22667	23044	23422	24556	24933	25311
52	22726	23111	23496	23881	24267	25422	25807	26193
53	23556	23948	24341	24733	25126	26304	26696	27089
54	24400	24800	25200	25600	26000	27200	27600	28000
55	25259	25667	26074	26481	26889	28111	28519	28926
56	26133	26548	26963	27378	27793	29037	29452	29867
57	27022	27444	27867	28289	28711	29978	30400	30822
58	27926	28356	28785	29215	29645	30933	31363	31793
59	28844	29281	29719	30156	30593	31904	32341	32778
60	29778	30222	30667	31111	31556	32889	33333	33778

Circular Functions.

Second quadrant:

$\sin A = \cos (A - 90)$ $\quad \cos A = -\sin (A - 90)$

$\tan A = -\cot (A - 90)$

Third quadrant:

$\sin A = -\sin (A - 180)$ $\quad \cos A = -\cos (A - 180)$

$\tan A = \tan (A - 180)$

Fourth quadrant:

$\sin A = -\cos (A - 270)$ $\quad \cos A = \sin (A - 270)$

$\tan A = -\cot (A - 270)$

Right Triangle.

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

$c^2 = a^2 + b^2$ Area $= \frac{1}{2} ab$, where c is the hypotenuse.

Oblique Triangle.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \tfrac{1}{2}(A + B) \quad \tfrac{1}{2}(A + B) = 90 - \tfrac{1}{2} C$$

$$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$$

$$B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)$$

$$c = (a + b) \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}}, \text{ where } s = \tfrac{1}{2}(a + b + c)$$

$$\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc} \qquad \text{vers } A = \frac{2(s-b)(s-c)}{bc}$$

$$\text{Area} = \tfrac{1}{2} ab \sin C = \sqrt{s(s - a)(s - b)(s - c)}.$$

General Formulas.

$$\sin A = \frac{1}{\text{cosec } A} = \sqrt{1 - \cos^2 A} = \tan A \cos A$$

$$= 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \text{vers } A \cot \tfrac{1}{2} A$$

$$= \sqrt{\tfrac{1}{2} \text{ vers } 2A} = \sqrt{\tfrac{1}{2}(1 - \cos 2A)}.$$

$$\cos A = \frac{1}{\sec A} = \sqrt{1 - \sin^2 A} = \cot A \sin A$$
$$= 1 - \text{vers } A = 2\cos^2 \tfrac{1}{2} A - 1 = 1 - 2\sin^2 \tfrac{1}{2} A$$
$$= \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = \sqrt{\tfrac{1}{2} - \tfrac{1}{2}\cos 2A}.$$

$$\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1} = \sqrt{\frac{1}{\cos^2 A} - 1}$$
$$= \frac{\sin 2A}{1 + \cos 2A} = \frac{1 - \cos 2A}{\sin 2A} = \text{exsec } A \cot \tfrac{1}{2} A.$$

$$\text{vers } A = 1 - \cos A = \sin A \tan \tfrac{1}{2} A = 2\sin^2 \tfrac{1}{2} A$$
$$= \text{exsec } A \cos A$$

$$\text{exsec } A = \sec A - 1 = \tan A \tan \tfrac{1}{2} A = \frac{\text{vers } A}{\cos A}$$

$$\sin \tfrac{1}{2} A = \sqrt{\frac{1 - \cos A}{2}} = \sqrt{\frac{\text{vers } A}{2}}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{1 + \cos A}{2}}$$

$$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$

$$\tan \tfrac{1}{2} A = \frac{\tan A}{1 + \sec A} = \text{cosec } A - \cot A = \frac{1 - \cos A}{\sin A}$$
$$= \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\text{exsec } \tfrac{1}{2} A = \frac{1 - \cos A}{1 + \cos A + \sqrt{2\,(1 + \cos A)}}$$

$$\text{exsec } 2A = \frac{2 \tan^2 A}{1 - \tan^2 A}$$

$$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$$
$$\cos (A + B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin A + \sin B = 2 \sin \tfrac{1}{2} (A + B) \cos \tfrac{1}{2} (A - B)$$
$$\sin A - \sin B = 2 \cos \tfrac{1}{2} (A + B) \sin \tfrac{1}{2} (A - B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2} (A + B) \cos \tfrac{1}{2} (A - B)$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2} (A + B) \sin \tfrac{1}{2} (A - B)$$
$$\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$
$$= \sin (A + B) \sin (A - B)$$
$$\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$$
$$\tan A + \tan B = \frac{\sin (A + B)}{\cos A \cos B}$$
$$\tan A - \tan B = \frac{\sin (A - B)}{\cos A \cos B}.$$

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13163	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97633	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74352	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

′	4°		5°		6°		7°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	8.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

′	8°		9°		10°		11°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

′	12°		13°		14°		15°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3 60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

′	16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

′	20°		21°		22°		23°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40630	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2 48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

′	28°		29°		30°		31°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	61°		60°		59°		58°		

′	32°		33°		34°		35°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50328	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	57°		56°		55°		54°		

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

′	40°		41°		42°		43°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1 11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

′	44°		′	′	44°		′	′	44°		′
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	Cotang	Tang	′	′	Cotang	Tang	′	′	Cotang	Tang	′
	45°				45°				45°		

FIELD BOOK

FOR

RAILROAD SURVEYING

BY

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PREFACE.

THIS book is an outgrowth of the notes which have been prepared from time to time to supplement the text-books used in railroad surveying at Cornell. They were enlarged to somewhat their present shape some three or four years ago and have since been used in mimeographed form as a text; the Transition Curve, with one or two additional tables, serving as the field book.

And while an apology is due for adding to the large list of books on this subject already available, it is felt that a book more adapted to the needs of the student without neglecting the requirements of practice would meet a real need. With this in view an effort has been made to bring out practical field methods and to give representative problems, leaving much of the work and most of the special problems for the student. Also, as far as practicable, all matter not directly connected with railroad surveying has been excluded. This has resulted in a book of convenient size for the pocket when combined with the Transition Curve, or Part II.

Long field computations requiring logarithms are unnecessary. For office computations, five-place logs are sufficient for all railroad field problems. Those by Gauss and by Hussey are the most convenient of any known to the authors.

CORNELL UNIVERSITY, ITHACA, N. Y., *October*, 1909.

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RAILROAD SURVEYING.

INTRODUCTION.

"Railroad Engineering is the art of making a dollar earn the most interest." — *E. H. McHenry in the Northern Pacific Rules.*

1. Railroads. — Railroads are commercial enterprises, their special business being the manufacture and sale of transportation. That is, they are built to earn profits for their owners, like all other manufacturing plants.

The life of all business projects may be divided into three more or less definite periods:

1. Investigation and design (called location in the case of a railroad), in which the feasibility of the plan and its value as an investment are studied and, if the outlook is satisfactory, plans are made for construction.
2. Construction.
3. Operation.

Capital is invested during the first two periods and earnings are expected during the third, — when the net income equals the gross earnings less operating expenses and fixed charges.

2. Railroad Engineering.—Railroad Engineering deals with all three periods and its general problem is to make the net income as large as possible.

The gross earnings are limited by the amount of transportation actually sold, *i.e.*, by the amount of traffic. The principal factors affecting this, assuming efficient operation, are the number and size of the towns served and the character of their inhabitants; the extent of the agricultural, mining and manufacturing industries along the line; its accessibility to the traffic and its position with respect to competing lines. It is obvious that these practically limit the earning capacity of a line and depend upon its location.

The operating expenses are also affected by the location, as the cost of carrying traffic will vary with the gradients, distance, curvature and rise and fall over which it must be taken,

and the cost of maintenance will vary with the character of the construction and of the ground on which it is constructed.

The fixed charges consist principally of taxes and interest on cost of construction, which again depend upon the location. Thus improvements of gradient or alinement for the purpose of decreasing operating expenses may increase the fixed charges (interest) by a larger amount, or vice versa.

3. Railroad Location. — The above brief discussion of the interdependence of the three periods and of the items affecting the net income indicates not only the complexity of the problem confronting the railroad engineer but also the supreme importance of the "location." The Locating Engineer must be versed in construction and operation in order to be able to determine the effect of his work on the net income.

Even if the broader questions of traffic are studied by the higher officials and the termini and most important intermediate controlling points are fixed, he must still know how much it is worth to his line to decrease gradient and to reduce distance, curvature and rise and fall, as well as the effect of differences in location on the cost of that portion of maintenance which is independent, or nearly so, of the traffic.

4. Railroad Surveying. — It will be impossible, however, to discuss even these minor questions within the scope of this book, so they will be left to be taken up in the work on Railroad Economics, to which subject they properly belong.

Attention will then be confined to Railroad Surveying, which may be defined as the treatment of surveying operations incident to the location, construction and operation of a railroad.

5. First Principles. — Railroad Surveying has much in common with other surveying, but we have seen that it is only a small part of the whole field of Railroad Engineering. Only a few of the more elemental principles of the latter will therefore have to be discussed with the surveying. The most important principle will be given here, as it should be learned at the outset of one's railroad training and never be forgotten or neglected.

This first principle recognizes the fact stated in the first paragraph and so aptly put in the quotation heading this Introduction. This "art" must begin with the survey and it demands economical work. Economical survey work requires

thought and careful planning to make every move count towards securing the desired results, concentrating on the essential, and slighting the nonessential when economy will result thereby.

Gross blunders and mistakes are best guarded against by avoiding haste, by constantly keeping in mind a broad general view of the problem and by applying the criterion of reasonableness to all results. Thus results plainly impossible should be rejected at once and those which appear unreasonable should be carefully scrutinized before acceptance.

Uniform general methods are desirable as tending to economy and freedom from mistakes, but the engineer should be ready to adapt his methods to the local conditions whenever economy will result thereby.

The accuracy of the work should be as great as is compatible with cost, that is, it should be increased up to the limit at which it begins to appreciably increase the cost; beyond this limit it should be increased only as actually required in order to secure the results needed. This would exclude inaccuracy due to poor methods, lack of skill, or carelessness, as these do not tend to reduce cost. On the other hand, it is easy to waste time and thought on the unimportant details, to the neglect of the important ones and especially of the engineering considerations upon which the survey depends.

6. General Field Methods. — Direct horizontal measurements are usually most convenient for a railroad survey, as they allow of placing the stakes which mark the stations at the usual and regular distance of 100 feet; it being common practice in railroad work to express distances in stations, or units of 100 feet. On level or uniformly sloping ground, the odd stations are sometimes omitted, thus leaving stakes at the even stations only, or 200 feet apart.

On sharp curves, on rough ground and for instrument points, intermediate stakes, called plusses, are frequently required. A plus of a half station length or of some multiple of 10 feet is more convenient for plotting and computation and should be taken if suitable. The front tapeman stops at the plus with his flagpole and the zero end of the tape, and is alined by the transitman. This leaves the rear tapeman, who should be held responsible for all tape readings, to note the plus. He can remain at the station and hold the end of the tape for the

next station if the line is straight, or he can come up to the plus and hold the remainder of the 100 feet on the tape to insure the proper distance between stations.

Each transit point should have a hub and a guard stake. The hub is driven nearly flush with the ground and the point accurately marked by a tack. Both line and guard stakes are marked with the number of the station or station and plus at which they are set. The guard stake should be driven about one and a half feet to the left of the hub, facing it and inclined slightly towards it to prevent rapid weathering. Line stakes should face backward and be vertical or inclined backward as directed by the engineer in charge. The numbers on the stakes should all read downward.

In measuring on rough or sloping ground, the horizontality of the tape is estimated by the tapeman. It is well to sight over the compass box of the instrument in passing and note where its horizontal plane cuts the ground or other convenient object. This levels up the tapeman's horizon and enables him to judge more accurately when the tape is horizontal. It should be remembered that a 100-foot steel tape when suspended under a 10- to 12-pound pull will project about 0.1 foot shorter than its true length, and this allowance should be made in setting the front stake if the tape is used normally flat or suspended in 50-foot lengths. Reliable taping on rough or rolling ground can only be secured by occasionally watching the tapemen and correcting their methods if found defective. On level ground the tape should be flat, with the ends on the ground. In fact the tape should always be kept as low as possible, *i.e.*, with at least one end on the ground unless intervening obstacles prevent.

If the ground is too steep for 50-foot lengths, the rear tapeman should use a hand level for reliable results. He should use a plumb bob, while the front tapeman can use his flagpole. This pole should be $2\frac{1}{2}$ inches wide, with a level for plumbing to secure accurate results in taping and pointing; otherwise he should take advantage of a tree or house corner at right angles to the line to check his estimate of verticality in taping.

7. Accuracy.—As will be seen later, small errors in alinement cause inappreciable errors in distance, so that it is useless to spend time in alining the regular line stakes closer than an inch. The hubs, however, must be accurately alined and

pointed. It is the practice of many engineers to double-center all hubs, and of others to double-center hubs on long curves or long tangents to eliminate errors of collimation.

Practice will soon teach one the care required to obtain a certain degree of precision. The judgment will, however, be aided by the following:

The accuracy desirable in setting over points, plumbing poles, etc., *i.e.*, in centering, is such that the errors from these causes shall not be greater than the errors of reading the vernier. This is easily determined by remembering that

$$\sin 1' = .000291,$$

which amounts to about 0.03 foot per 100. Thus an error of 0.0075 foot, or about $\frac{1}{16}$ inch, would cause an error of $\frac{1}{2}$ minute on a 50-foot sight. Hence to run to half minutes with 50-foot sights requires care in setting the instrument and very great care in sighting at the points if the rods cannot be seen nearer to the ground than 4 or 5 feet. In fact flagpoles with cross levels and well-defined sight lines are essential for rough ground in working to minutes unless the sights can be made longer than 50 feet. The maximum length of sight in pointing should not exceed 1200 feet. With fairly smooth country $\frac{5}{8}$-inch steel rods or $1\frac{1}{8}$-inch steel-pointed wooden rods 6 to 8 feet long give good results and are much used.

By remembering that $\sin 1' = 0.00029$ and $\sin 1° = 0.01745$, or about 0.03 and 1.75 feet per station, respectively, many

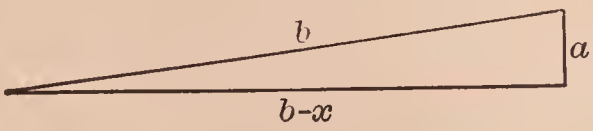


Fig. 1.

field computations can be made without reference to tables, or even the use of paper and pencil, which will greatly facilitate the passing of obstacles, making corrections, estimating deflection angles, etc.

The formula for correction to distance due to error of alinement, or to inclination, is derived as follows: In Fig. 1

$$\begin{aligned} b^2 &= a^2 + (b - x)^2 \\ &= a^2 + b^2 - 2bx + x^2. \end{aligned}$$

Solving, $$x = \frac{a^2}{2b - x}, \text{ or}$$

$$x = \frac{a^2}{2b} \text{ (nearly)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For $a = 0.1\ b$ the error due to neglecting x in the denominator of the second member is only about 1 in 100 000.

For a 100-foot tape an error of one foot in alinement or in level would make an error of $\frac{1}{200}$ of a foot in distance, while for a 50-foot tape the error would be twice as great in a tape length or four times as great per 100 feet.

CHAPTER I.

Simple Curves.

8. Simple Curves.—Formerly the alinement of a railroad as laid out was made up of simple curves and tangents, the former being arcs of circles and the latter straight lines tangent to them. The portions of the straight lines between

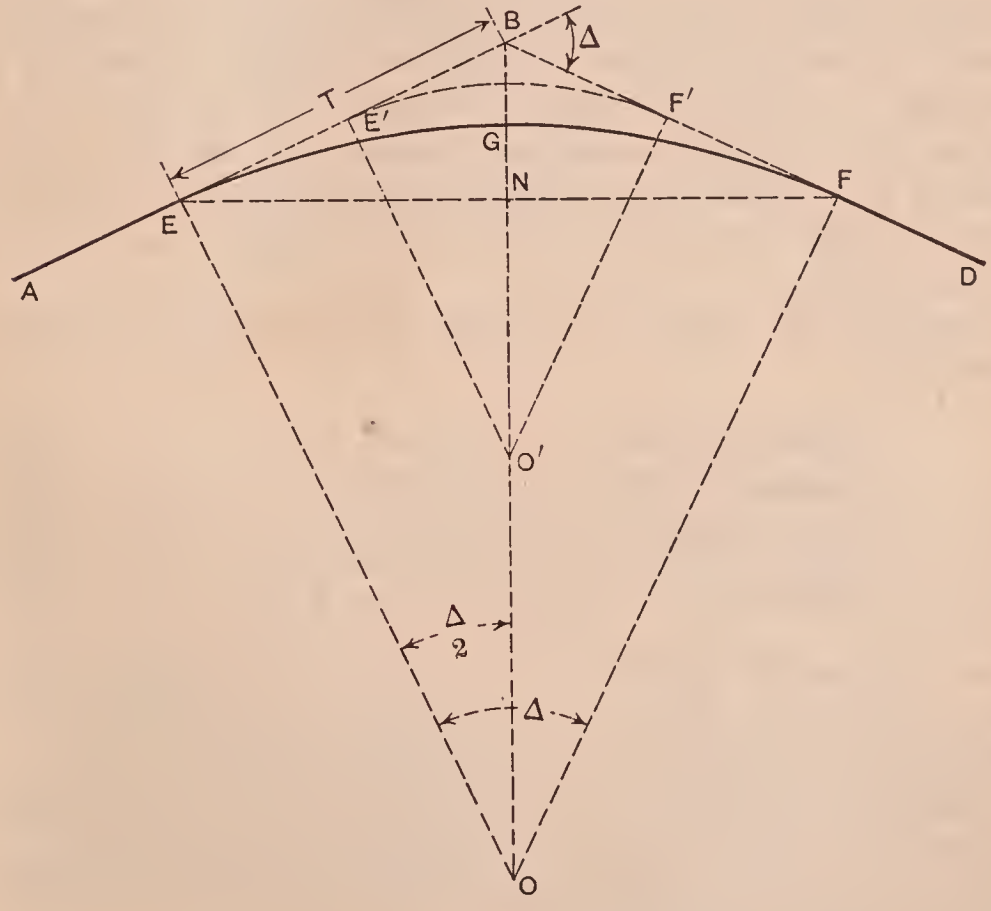


Fig. 2.

the tangent points of adjacent curves are used for the alinement, not the semi-tangents BE and BF of Fig. 2, which are called tangent distances and are generally used as auxiliary lines in running the curves. At present the practice is quite common to change the curvature gradually from the straight line to the simple curve by means of a transition curve or spiral. This modification will be considered later.

In changing the direction from one tangent to the next, as from AB to BD in Fig. 2, the radius of the curve is usually fixed by external conditions; if not, it would have to be assumed to make the problem definite, that is, the turn could be made with a curve of any radius, as by curve EF with radius EO or by $E'F'$ with radius $E'O'$.

For curve EF in Fig. 2, E and F are the tangent points, and, assuming the curve to be run from left to right or from E to F, the point E is called the point of curve, PC, and the point F the point of tangent, PT; the intersection of the tangents, B, is called the point of intersection, PI; the deflection angle from one tangent to the next is called the intersection or central angle, Δ; the distance from the PI to the tangent point (BE or BF) is called the tangent distance, or simply the tangent, T; the perpendicular distance from the PI to the curve, BG, is called the external distance, E; the chord EF joining the tangent points is called the long chord, C, and the distance of its center from the curve, NG, is called the middle ordinate, M.

Denoting the radius of the circle by R we have by trigonometry:

$$\text{Tangent distance, } T = R \tan \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{External distance, } E = R \operatorname{exsec} \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{Middle ordinate, } M = R \operatorname{vers} \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad (4)$$

$$\text{Long chord, } C = 2R \sin \tfrac{1}{2}\Delta \quad . \quad . \quad . \quad . \quad (5)$$

9. Degree of Curve. — Since the above functions of a curve depend directly upon the radius, it seems natural to designate a curve by its radius, and this is the English practice. In America, however, a curve is designated by its degree for greater convenience in laying out by the method of deflections, except for very sharp street railroad curves, where the English system is frequently followed.

The degree of curve, D, is the central angle subtended by a chord of 100 feet. This would give, Fig. 3,

$$R = \frac{50}{\sin \frac{1}{2} D}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which makes the radius vary inversely as D for small angles. For a 1° curve, $R = 5729.65$; for a 10°, $R = 573.69$; for a

20°, $R = 287.94$, etc; while the corresponding arcs, or lengths of track between 100-foot stations, are, in feet, 100.001 100.127, 100.510, etc.

It is customary and convenient to tabulate the tangents and externals of a 1° curve as found by (2) and (3) and to find those for other curves by dividing by D. These quotients require corrections which increase with D and with Δ. On account of this difficulty and because of sharp curves being so

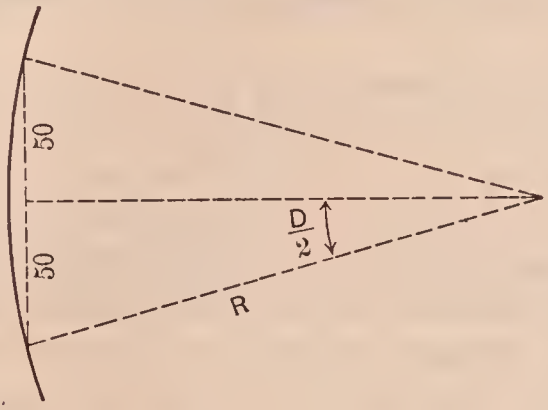


Fig. 3.

much longer than their nominal or recorded lengths, it has been proposed to make the degree of curve the central angle subtended by an arc of 100 feet and to use a table of chord corrections in the field, shortening the chord as D increases so as to give 100-foot arcs. This method has not come into extended use.

Another method suggested by A. M. Wellington has come into more general use. It consists in using shorter chords for the sharper curves, 50-foot for curves from 8° to 16°, 25-foot from 16° to 32°, and 10-foot for sharper curves, — so that the chords will closely approximate the arcs. The radius of the 1° curve is changed from 5729.65 to 5730, while that for a $D°$ curve is $5730/D$. This method has been followed in the Transition Curve of Part II. It is accurate enough for all field-work except possibly running track centers, for which corrections are given on page 39 of Part II. Sharp curves require stakes closer together than 100 feet to properly define them, so that the short chords seldom require useless labor.

Accepting this modified definition,

$$\text{Length of curve in stations, } L = \frac{\Delta}{D} \quad . \quad . \quad . \quad (7)$$

$$\text{Radius in feet, } R = \frac{5730}{D} \quad . \quad . \quad . \quad (8)$$

The radius of a 1° curve can be easily remembered by association with the number of feet (5280) in a mile; the first and last digits are the same, as also the sum of the other two.

Tangents and externals for a 1° curve are given in Table VII of Part II with Δ as argument; those for a $D°$ curve are readily found by dividing by D. The radii are given in Table V; if D is integral, as will usually be the case, they are as readily found for the flatter curves by simple mental division.

10. The Deflection Method for laying out Curves. — The methods of laying out curves are simply those of drawing arcs of circles on the ground. When the radius is small, say less than 100 feet, the arc may be swung in from its center, using the tape. This is sometimes done in street railroad work, but ordinarily the deflection method is used on account of the impracticability of using the radius. This method depends upon the theorem that for a circle the angle between a tangent and a chord or between two chords is measured by one-half the intercepted arc and that angles at the center are measured by the intercepted arcs. Thus with the transit at any point of the curve the deflection for 100 feet (measured in whole, halves, quarters or tenths of a tape length according to the sharpness of the curve) will be $D/2$ and proportionately for other distances.

Thus, in Fig. 4, suppose that the tangent has been located from the left to the PI and that Δ has been measured. Assume a degree of curve, D, that will meet the conditions imposed. Then find the radius from Table V or by dividing 5730 by D, and the tangent distance, T, by (2), or more conveniently by dividing the tangent for a 1° curve, T_1, from Table VII by D. Subtract T from the stationing for PI for the PC and measure T forward from the PI for the PT. Then take the transit back to the PC, assumed as sta. 162 + 25, and deflect from the tangent as follows:

Sta. 163, or A, .75 of $\frac{1}{2}\ D$.

Sta. 164, B, .75 of $\frac{1}{2}\ D + \frac{1}{2}\ D$.

Sta. 165, C, .75 of $\frac{1}{2}\ D + D$.

Sta. 166, D, .75 of $\frac{1}{2}\ D + \frac{3}{2}\ D$.

Sta. 167, E, .75 of $\frac{1}{2}\ D + 2\ D$.

Sta. 167 + 30 PT, .75 of $\frac{1}{2}\ D + 2\ D + 0.3$ of $\frac{1}{2}\ D = \frac{1}{2}\ \Delta$, as the deflection for the PT is always one-half the central angle. This checks all of the deflections, and these notes should be written up before running the curve.

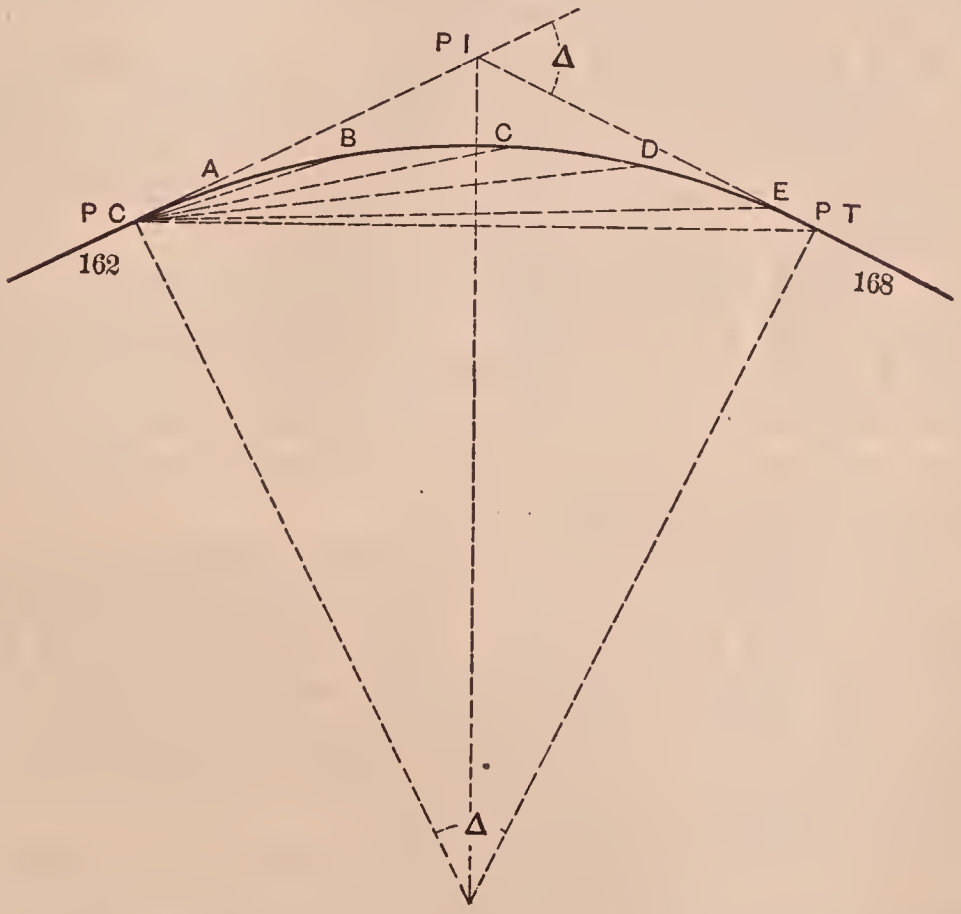


Fig. 4.

A set of deflections made up in this way may be used with the instrument at any point on the curve. Each deflection or vernier reading should be associated with the station to which it belongs, and it can be used for that station so long as the instrument is set up anywhere on the curve. This will require the vernier for the backsight to be set at the reading opposite the station on which the backsight is taken. The

reading opposite the station where the instrument is set up will give the tangent to the circle at that point.

Obstructions often require the instrument to be moved up one or more times in running a curve, while if the central angle is large, accuracy will require a new set-up as soon as the arc described by the front end of the tape revolving around the last station as a center does not give a large-angled intersection with the line of sight from the instrument. These new instrument points will have no effect on the curve notes or vernier readings made up once for all as indicated above, unless plus stations are used, in which case the extra vernier readings are simply interpolated.

Many engineers make up a new set of vernier readings for each intermediate instrument point on the curve, starting from zero on the new tangent. While this system has some advantages, it is believed that the method outlined above will save labor and reduce the danger of mistakes.

11. Form of Field Notes. — The form of transit notes shown in Fig. 5 is convenient. The ⊙'s in the first column show the hubs or transit points on the line. They indicate where the line can be accurately picked up for further instrument work, and to quite an extent the character of the work, and they should always be given.

The computations for the 6° curve are given in full in the column of Curve Data and no further explanation is required except to state that the 167 + 51.6 opposite the *PT* is the stationing around the tangents and is found by adding the tangent, *T*, to the *PI*. The track is to be laid around the curve, so the stationing 167 + 30 by the curve is the one which will govern in running the next tangent.

T_1, the tangent for a 1° curve for $\Delta = 36° 18'$, is taken from Table VII as already indicated.

The bearings noted are read from the needle for each tangent, and they should always be given as a rough check on the work and for comparison with those of property and other required lines. The bearings shown in brackets are computed from the Δ's of the curves, starting from the first or reference tangent. Table V, giving deflection angles in minutes per foot of chord, may be conveniently used when the sub-chord deflection cannot readily be computed mentally. The date, party and usually the weather conditions, should be given at the

beginning of the day's work. If the work is not a continuation of that immediately preceding in the record it should be described. Under this latter head should be given the title of the problem if it is problem work.

12. Typical Curve Problem.—In the typical or standard curve problem one tangent has been located and station stakes set and the other tangent has been picked out on the ground.

Set up at the *PI*, measure Δ, and note the bearing of the front tangent; while the tapemen are chaining for the *PT* look up T_1 in Table VII, divide by *D* and give the resulting tangent length to the rear tapeman for locating and pointing the *PT* hub; subtract *T* from *PI* for the *PC*; divide Δ by *D* for length of curve, *L*, which add to the stationing of the *PC* and give the result to the stakeman, for marking the guard at the *PT*; check the backsight and give the *PC* stationing to the tapemen so that they can put in the *PC* hub and guard; complete the curve notes, checking the *PT* through the vernier readings as indicated in §11; set up on the *PC* and backsight on the tangent with vernier at zero. By taking up the work in this order the transitman can keep his party busy and have the notes ready for running the curve as soon as the *PC* is set.

If the *PT* is visible from the *PC* it is well to turn one-half Δ and note the check upon the *PT*, as this checks the equality of the tangents and the value of Δ. In running the curve, the last sub-chord should check the computed value, and the line of sight should check the tack on the *PT* hub. The discrepancy, if within allowable limits, is usually left in the curve, the front tangent starting off at the *PT* from a back sight on the *PI*. The same direction should of course be obtained by backsighting to the last hub on the curve and turning to the vernier reading ($\frac{1}{2}\ \Delta$) for the *PT*.

As already stated, as many intermediate instrument stations as necessary may be used on the curve without disturbing the notes except to enter the ⊙'s and also the extra vernier readings in case full stations cannot be utilized. The guards at the ends of curves should be marked to show the alinement, as *PC* 3° *R*, *PT*, etc.

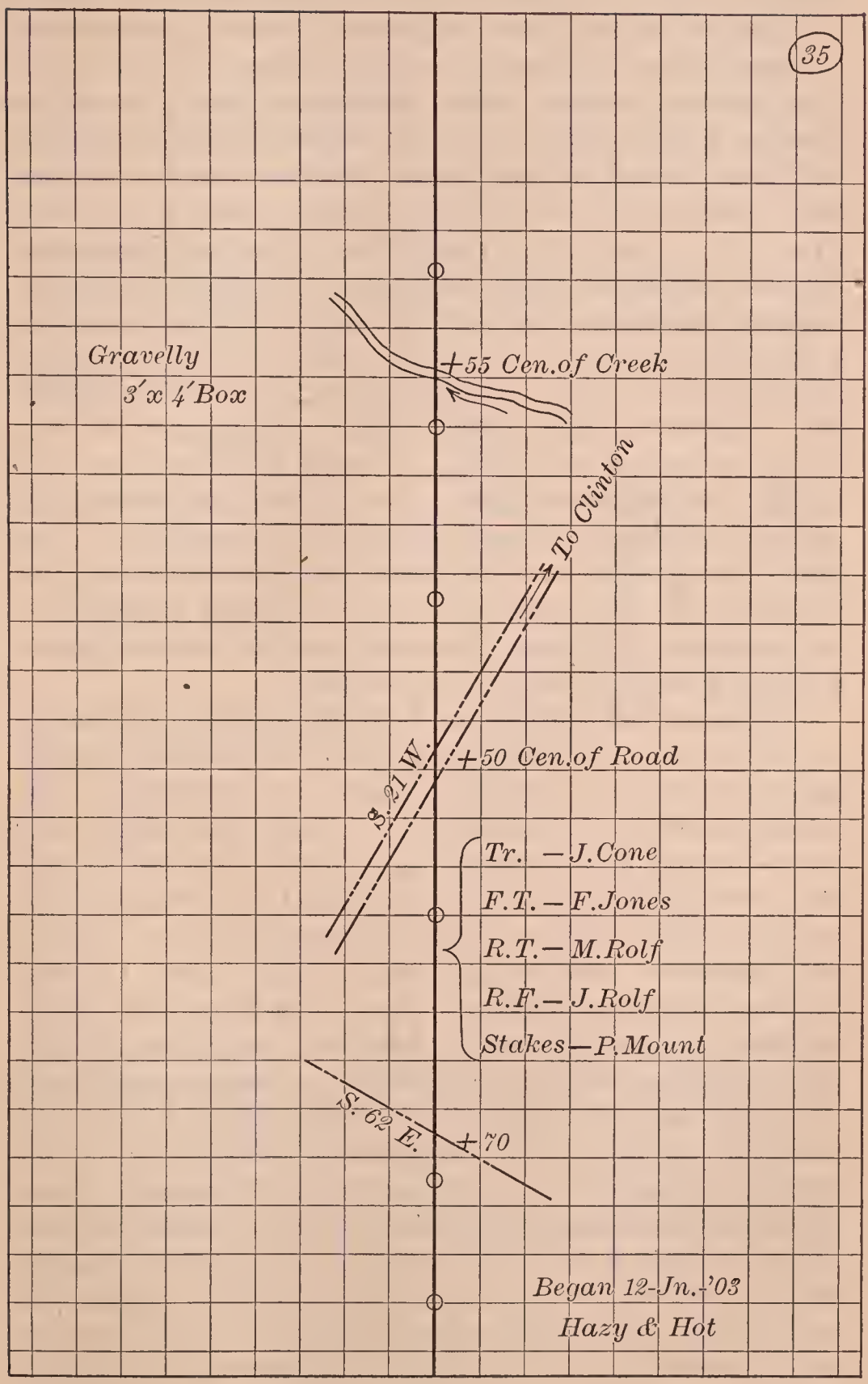
If the *PI* is not given, two stakes may be pointed on the rear tangent not over 100 feet apart, and preferably less than 20, so as to include the *PI* between them. The transit may

TRANSIT

Sta.	Point.	Curve or Bear.	Vern.	Curve Data	
1			2–00		
+60.0 ⊙	PC	10°R			
170					
9 ⊙					
8					
+30.0 ⊙	PT	S 6-20 W	18-09	= 167	$+\frac{5}{4}$1.6
7		[6–28]	17–15		
				Δ =	36–18
6			14–15	$L=\frac{36.3}{6}$	=6.05
				R =	955
165			11–15	T_1 =	1878.4
				T =	313.1
4 ⊙			8–15	PI =	164+38.1
				PC =	161+25.0
3			5–15	PT =	167+30.0
2			2–15		
+25.0 ⊙	PC	6°R	0		
1					
160 ⊙		S 30–10 E			
		[29–50]			

Figure

BOOK


35
Gravelly
3′x 4′Box
+55 Cen.of Creek
To Clinton
S. 21 W.
+50 Cen.of Road
Tr. —J.Cone
F.T. —F.Jones
R.T.— M.Rolf
R.F.— J.Rolf
Stakes—P.Mount
S. 62 E.
+70
Began 12-Jn.-'03
Hazy & Hot

then be set over the nearer of the two points fixing the front tangent and the *PI* found at the intersection of the line of collimation for the front tangent with the tape stretched between the two stakes on the rear tangent.

In general, to solve a field problem, a sketch should be made, as nearly to scale as may be, putting in the known data and then proceeding step by step to determine the required data upon the supposition that it is being done by plotting. After this is done, follow the plotting steps by trigonometric relations or formulas which will furnish a basis for the required numerical solution. It is much easier to follow relations graphically, and it is usually possible to secure numerical results for any problem in field geometry which can be plotted. With this habit once acquired, one is independent of field books except for the tables, and much better equipped for rapidly and accurately seeing the relations and solving the problems arising in railroad location or in track work. On this account only a few of the special problems which come up in connection with simple curves and obstacles in alinement will be given, and these to bring out methods rather than on account of any difficulty in their solution.

Long numerical solutions in the field should be avoided so far as possible. The delays occasioned are expensive, demoralizing to the party and injurious to its reputation. Frequently much of the solution can be done graphically with the instrument and party with a saving of time and reputation; at other times data can be gathered for an office solution and other work taken up for the balance of the day. Study and experience soon enable one to approximate the best obtainable results in fitting a line to the ground by simple methods and simple geometric relations, leaving the difficult special problem for yards, city streets, or rugged topography.

13. To lay out a Curve with Inaccessible *PI*.—From a suitable point on the first tangent run a traverse over convenient ground to reach a point on the second tangent. These points on the tangents should be near the desired tangent points, or at least not much nearer the *PI* than the tangent points. Assume the first tangent as a meridian or axis of X and compute the coördinates of the points on the traverse with reference to this, using the starting point as the origin. Divide the ordinate of the station on the second tangent by

sin Δ for the distance of the point from the PI and multiply this ordinate by cot Δ for the projection of this distance on the axis. Subtract this projection from the x coördinate of the point for the distance of the point on the first tangent from the PI. Compare these distances from the PI with the tangent of the required curve for the distances to move from these points on the tangents to the PC and PT. After setting these the remainder of the problem will be the same as given in §12.

EXAMPLE. — Given at sta. 184, deflection 4° 18′ left; sta. 186 + 50, 2° 10′ right; sta. 189, 6° 25′ left; sta. 192, 1° 18′ left, coinciding with the second tangent. Required the data for locating a 1° curve.

Solution.

Station	Deflection			Dist.	x	y
184	4°	18′	L	250	249.30	18.74
186 +50	2	8	L	250	249.83	9.31
189	8	33	L	300	296.67	44.60
192	9	51	$L = \Delta$			
Coördinates of sta. 192,					795.80	72.65

Distance of sta. 192 from $PI = \dfrac{y'}{\sin \Delta} = \dfrac{72.65}{.17107} = 424.7.$

Projected distance, $y' \cot \Delta = 72.65 \times 5.75941 = 418.42.$

Sta. 184 from PI, $795.80 - 418.42 = 377.38.$

$PI \qquad = 187 + 77.4$

T_1 (Table VII) $= 493.7 = \quad 4 + 93.7$

$PC \qquad = 182 + 83.7$

$L = \dfrac{9.85}{1} \qquad\qquad 9 + 85$

$PT \qquad = 192 + 68.7$

The PT will be $493.7 - 424.7 = 69.0$ beyond the old sta. 192.

14. To lay out a Curve with Inaccessible PC. — Extend the tangent to a regular station near the obstruction as at A, Fig. 6, and pass by an equilateral triangle with sides one or more full stations long, or by a traverse or other method suitable for the conditions, and determine the PI and Δ. Find T and PC, and by comparison with CD decide upon the station number for G; subtract the PC from G in stations and multiply by D for α.

$$\left.\begin{aligned} DF &= T - R \sin \alpha \\ FG &= R\,(1 - \cos \alpha). \end{aligned}\right\} \qquad (9)$$

Set the PT and F while the instrument is at PI and set up at F, turn 90° from PI, set G and place a backsight hub H on FG produced some 200 feet, if FG is short. Set up at G, backsight on H with vernier at $90° - \frac{1}{2}\alpha$ and continue the curve from the notes made up as usual.

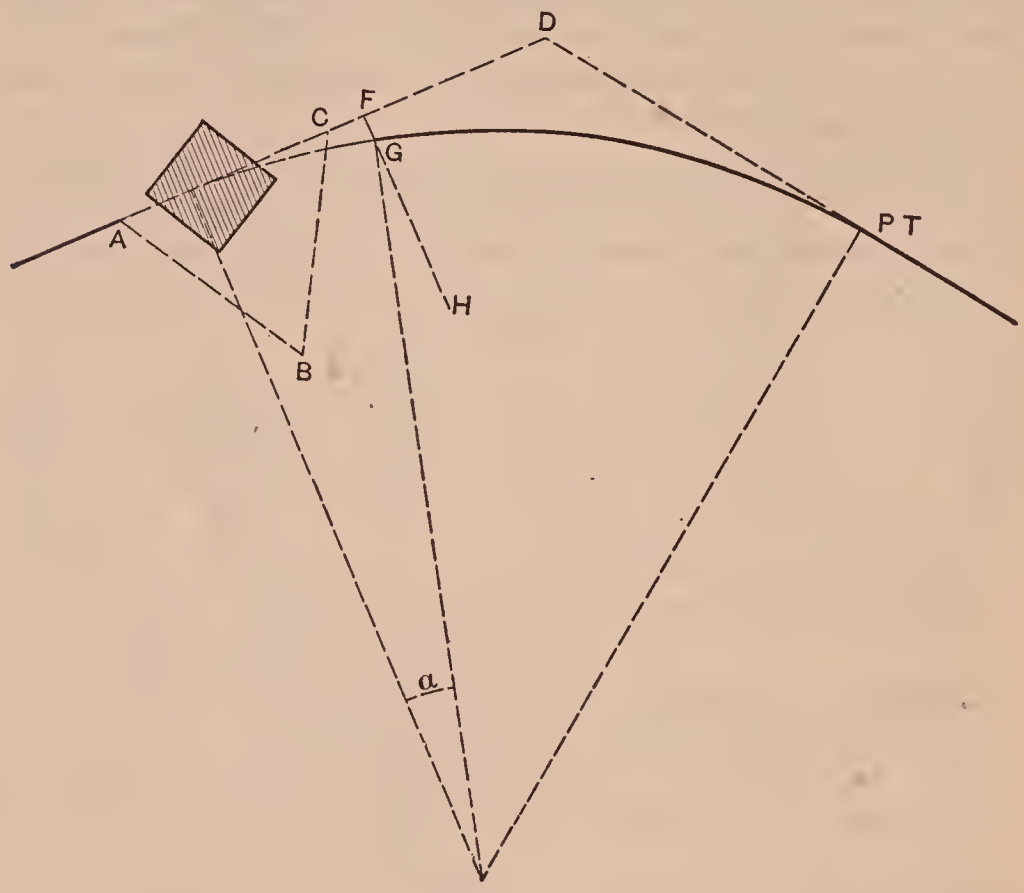


Fig. 6.

A sketch with the special data and computations should be shown on the right-hand page.

A simpler method would be to turn $90 - \frac{1}{2}\Delta$ from the backsight when at PI and measure the external, E, locating the central point of the curve.

The vernier for the backsight from this point would be $90° - \frac{1}{4}\Delta$; the regular curve notes would answer with an interpolated reading for this central point, but the first portion of the curve would have to be backed in and would not be checked.

Example. — Given: — sta. 185, 60° R for two stations; 120° L for two stations to sta. 187; sta. 187, 60° R; sta. 189 + 70, PI; $\Delta = 18° 20' R$.

Find the data for a 2° curve and the coördinates for a starting point on the curve free of the obstruction.

Table VII, $T_1 = 924.6$, $E_1 = 74.12$, $T = \frac{924.6}{2} = 462.3$;

$$E = \frac{74.12}{2} = 37.06; \; R = 2865.$$

$$PC = 185 + 07.7; L = \frac{18.33}{2} = 9 + 16.7;$$

$$PT = 194 + 24.4.$$

For sta. 187 on the curve; $\alpha = 1.923 \times 2 = 3° \; 50\frac{1}{2}'$.

PC to $F = 2865 \times \sin 3° \; 50\frac{1}{2}' = 2865 \times .06700 = 192.0$

$DF = 462.3 - 192.0 = 270.3$

$FG = 2865 \; (1 - \cos \; 3° \; 50\frac{1}{2}') = 2865 \times .00225 = 6.45$; giving the coördinates for sta. 187 of the curve.

15. To change a Curve so as to end in a Parallel Tangent. —

(*a*) by sliding the PC on the first tangent. If the PC slides along the first tangent the PT will slide the same distance parallel with the first tangent, so that in the triangle ABD, Fig. 7, for a perpendicular distance p between tangents the distance AD moved along the first tangent will be found from

$$AD = \frac{p}{\sin \Delta} . \qquad (10)$$

This distance AD through which the PC moves is to be measured backward if the new tangent is inside the old, as in Fig. 7, and forward if the new tangent is outside the old.

Shifting the position of the curve will change the transit notes to correspond with the stationing of the new tangent points.

Example. — Sta. 184 + 50, PC, 4° R; sta. 190, PT. Required the new tangent point and the vernier readings for running the curve if the new tangent is to come 10 feet inside the old.

By (10),

$$AD = \frac{10}{\sin 22°} = \frac{10}{.37461} = 26.7.$$

New alinement: 184 + 23.3, PC, 4° R; deflections, — 185, 1° 32′; 186, 3° 32′; 187, 5° 32′; 188, 7° 32′; 189, 9° 32′; 189 + 73.3, PT, 11° 00′.

(*b*) by changing the degree of the curve.

If the PC remains fixed, the new tangent T' would be less than the old by AD, Fig. 7, giving

$$T' = T \pm \frac{p}{\sin \Delta} . \tag{11}$$

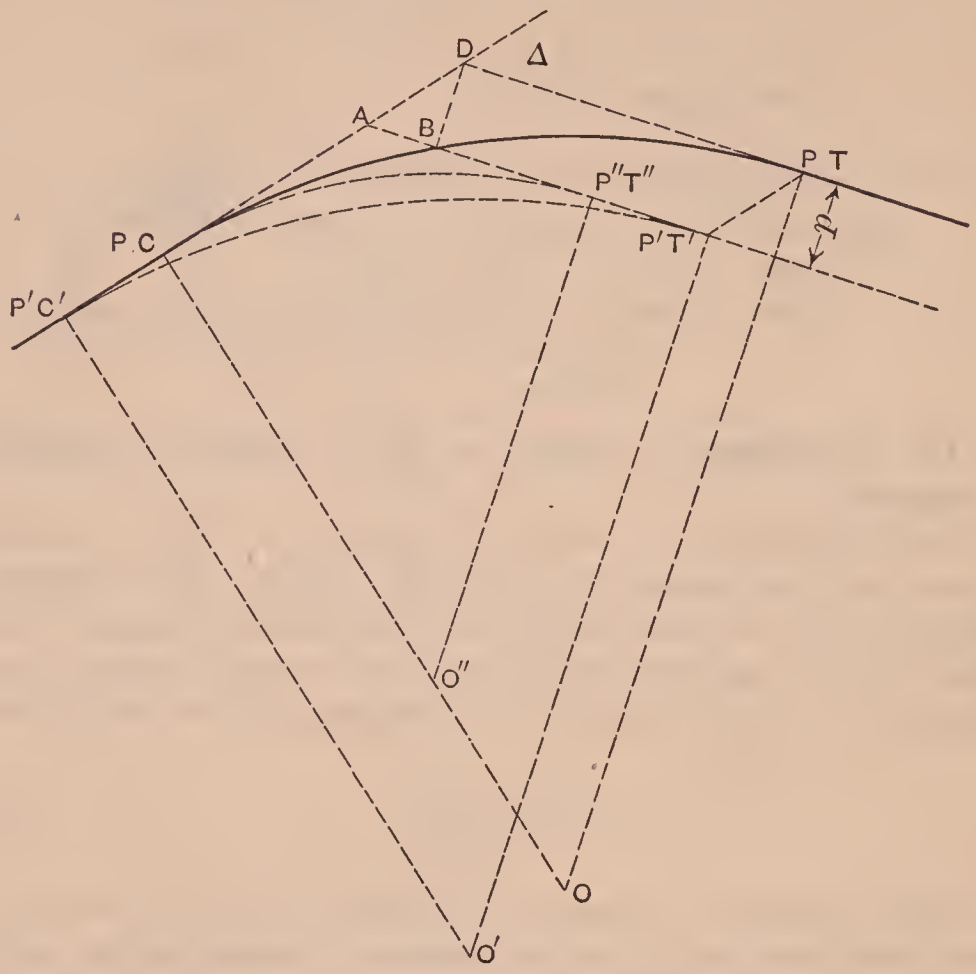


Fig. 7.

If the PT is to come opposite the old,

$$T' = T \mp p \cot \Delta, \tag{12}$$

where the upper sign is for the new tangent outside the old and the lower sign for the new tangent inside the old. Having T',

$$D' = \frac{T_1}{T'} . \tag{13}$$

16. To draw a Common Tangent to Two Given Circles.—This problem may arise in changing the location of part of a line or in joining up a new location to old track.

The curves are connected in the field by stopping the first at such a point that its tangent when extended will cut the other. The stationing for this intersection point is found and the deflection angle measured with one or more of the chords radiating from it. This supplies the data shown in Fig. 8.

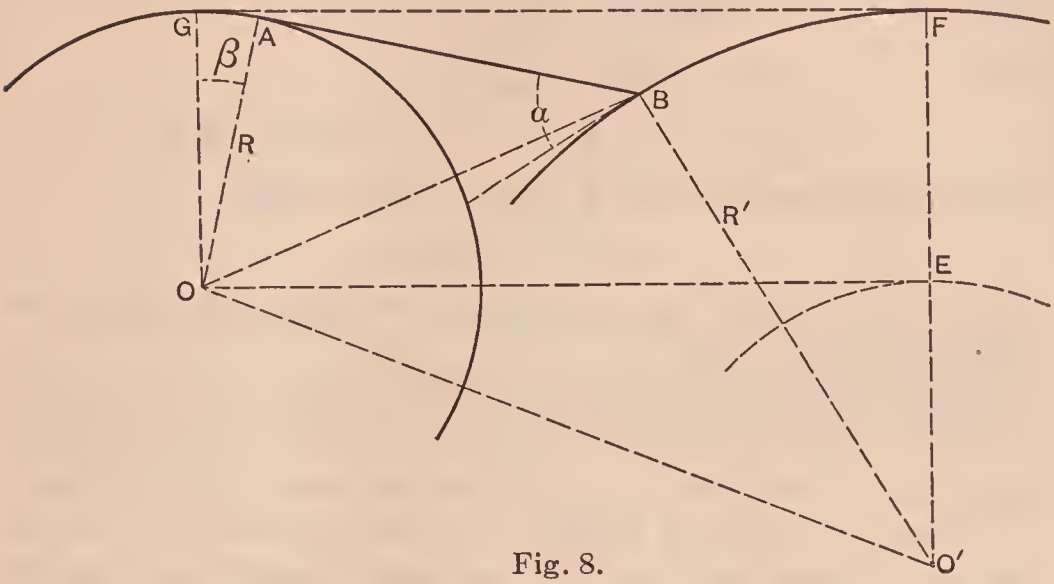


Fig. 8.

Thus AB is tangent to the first curve at A and intersects the second at B with α for the intersection angle.

In the right triangle ABO, Fig. 8,

$$\tan O_1 = \frac{AB}{R}, \qquad (a)$$

$$OB = \frac{AB}{\sin O_1}. \qquad (b)$$

In the triangle BOO',

$$\text{angle } B_2 = 90 + \alpha - (90 - O_1)$$
$$= \alpha + O_1.$$

OB and $O'B$, or R', are known.

Substituting for the oblique triangle with two sides and the included angle known,

$$\tan \tfrac{1}{2}(O_2 - O') = \frac{R' - OB}{R' + OB} \tan (90 - \tfrac{1}{2} B_2). \qquad (c)$$

$$O_2 = \tfrac{1}{2}(O_2 + O') + \tfrac{1}{2}(O_2 - O'),$$
$$O' = \tfrac{1}{2}(O_2 + O') - \tfrac{1}{2}(O_2 - O'),$$

$$OO' = R' \frac{\sin B_2}{\sin O_2}. \qquad (d)$$

With O' as a center and $R' - R$ as a radius describe a circle. A tangent to this circle through O will be parallel to the required tangent.

To find OE and the angle data:

In the right triangle OEO'

$$\sin O_3 = \frac{R' - R}{OO'},$$

$$OE = OO' \cos O_3,$$

$$\beta = 90 + O_3 - (O_1 + O_2).$$

The distance to move the PT on the first curve,

$$GA = \beta/D.$$

For the PC at F on the second curve, add OE to the stationing for the PT at G.

There are three other cases which are left for the student; one with the centers on the opposite sides of the required tangent and the other two with the flat curve coming first.

Unless a traverse is required in passing from A to B on account of obstructions, this problem can be more easily solved in the field by locating A by §21 so near the required PT that the angle between AB and GF will be so small that it may be converted into distance on the curve by dividing by D.

If a traverse is required (or available) to give the relative positions of the two curves, the problem is readily solved by computing coördinates, referred to a convenient axis, joining the curve centers and computing the necessary distances and angles as above.

Example. — The tangent at sta. 162, the PT of a 6° L curve intersects a 2° curve at sta. 168 with $\alpha = 4° 18'$ R. Required the PT of the 6° and the PC of the 2° for a common tangent. As this is an office problem the computation by logs will be given in full.

(a), $AB = 600$	2.77815	(b), $AB = 600$	2.77815
$R_6 = 955$	2.98000	$O_1 = 32° 08'$ sin	9.72582
$O_1 = 32° 08'$ tan.	9.79815	$OB = 1128.1$	3.05233

α,	4° 18′	(c), $R_2 - OB$, 1736.9	3.23977
O_1,	32 08	$R_2 + OB$, 3993.1 (AC)	6.39869
B_2,	36 26	$90 - \frac{1}{2} B_2$, 71° 47′ . . . tan	0.48266
		$\frac{1}{2}(O_2 - O')$, 52° 53′ . . tan	0.12112

O_2, 124° 40′; O', 18° 54′.

(d), R_2, 2865	3.45712	$R_2 - R_6$, 1910 ...	3.28103
B_2, 36° 26′ sin ..	9.77370	OO', 2068.7	3.31570
O_2, 124 40 sin (AC)	0.08488	O_3, 67° 24′ sin ...	9.96533
OO', 2068.7	3.31570		

$90 + O_3$,	157° 24′	O_3, 67° 24′ cos...	9.58467
$O_1 + O_2$,	156 48	OO', 2068.7.....	3.31570
β,	0 36	OE, 795.0.....	2.90037

$GA = \frac{0° \ 36'}{6} = 0 + 10.$ New $PT = 162 - (0 + 10) = 161 + 90.$

Tangent Distance $GF = OE = 7 + 95$; PC 2° $L = 169 + 85$.

17. Sub-Chords. — Referring to §9, it may be noted that if curves are computed and run on the basis of 100-foot chords a sub-chord length c' for a given deflection angle d would be found from (6),

$$c' = 2R \sin d, \tag{14}$$

with $R = 50/\sin \frac{1}{2} D$.

Or, if the sub-chord c' is given, the corresponding deflection angle d would be found from

$$\sin d = \frac{c'}{2R}. \tag{15}$$

These values will not differ appreciably from the nominal ones for curves flatter than 8°.

On the basis of shortening the chords as the degree of curve increases the true sub-chord would be

$$c' = 2R \sin d, \tag{16}$$

with $R = 5730/D$.

This will differ so little from the nominal length that no correction will be necessary except in cases of extreme accuracy in running track centers.

18. Metric Curves. — In working under the metric system it is customary to use a 20-meter tape and to take the central angle subtended by a tape length (for flat curves) as the degree of curve. This makes the radius of a metric curve to that of the foot curve of the same degree as 65.62 (the unit chord) is to 100 (the unit chord), or approximately as 2 to 3,

while there are but one-fifth as many units in the chord, 20 in place of 100, and hence in the tangent or any of the linear functions of the curve.

The tables for the English or foot system are thus available for the metric by simply dividing all linear functions by 5 D instead of by D.

The stations are numbered as if a 10-meter tape were used, leaving out the odd-numbered stations except when intermediates are required.

This gives one-fifth as many units in the curve functions and one-tenth as many in the station numbers as in the foot system, which is confusing at first, but the confusion soon disappears with use.

In following up the Wellington system for degree of curve it would be somewhat on the safe side to change to 10-meter chords for from 8° to 16° curves, 5-meter chords from 16° to 32°, and 2-meter chords for curves sharper than 32°.

19. Other Methods for laying out Curves. — Most of these are for use without a transit and are not capable of great accuracy except for short curves on smooth ground.

(*a*) by offsets from the tangents. This is a method by coördinates, using one of the tangents as an axis and the tangent point as an origin. In Fig. 9, A is the PC and B, D, E and F are regular stations on the curve.

Let $AC = n$, a fractional part of a station length. Then $AB = 1 - n$, and the angle

$$\left.\begin{aligned} E'AB &= \tfrac{1}{2}(1-n)D, \\ AB' &= AB\cos\tfrac{1}{2}(1-n)D, \\ BB' &= AB\sin\tfrac{1}{2}(1-n)D. \end{aligned}\right\} \qquad (17)$$

A tangent at B would make an angle of $(1 - n)D$ with the original tangent, and the chord BD would make an angle of $D/2$ additional.

Hence the angle between the chord BD and the tangent CE' is $(\frac{3}{2} - n)\ D$, and

$$\left.\begin{aligned} B'D' &= BD\cos(\tfrac{3}{2}-n)D, \text{ or} \\ B'D' &= 100\cos(\tfrac{3}{2}-n)D, \\ DD'' &= 100\sin(\tfrac{3}{2}-n)D. \end{aligned}\right\} \qquad (18)$$

Similarly,

$$\left.\begin{aligned} DE'' &= 100\cos(\tfrac{5}{2}-n)D, \\ EE'' &= 100\sin(\tfrac{5}{2}-n)D. \end{aligned}\right\} \qquad (19)$$

After passing the middle of the curve the offsets and distances can be found by working from the other tangent if more convenient.

On flat curves the offsets only need be computed and the chords can be used with them in locating the stations. Thus

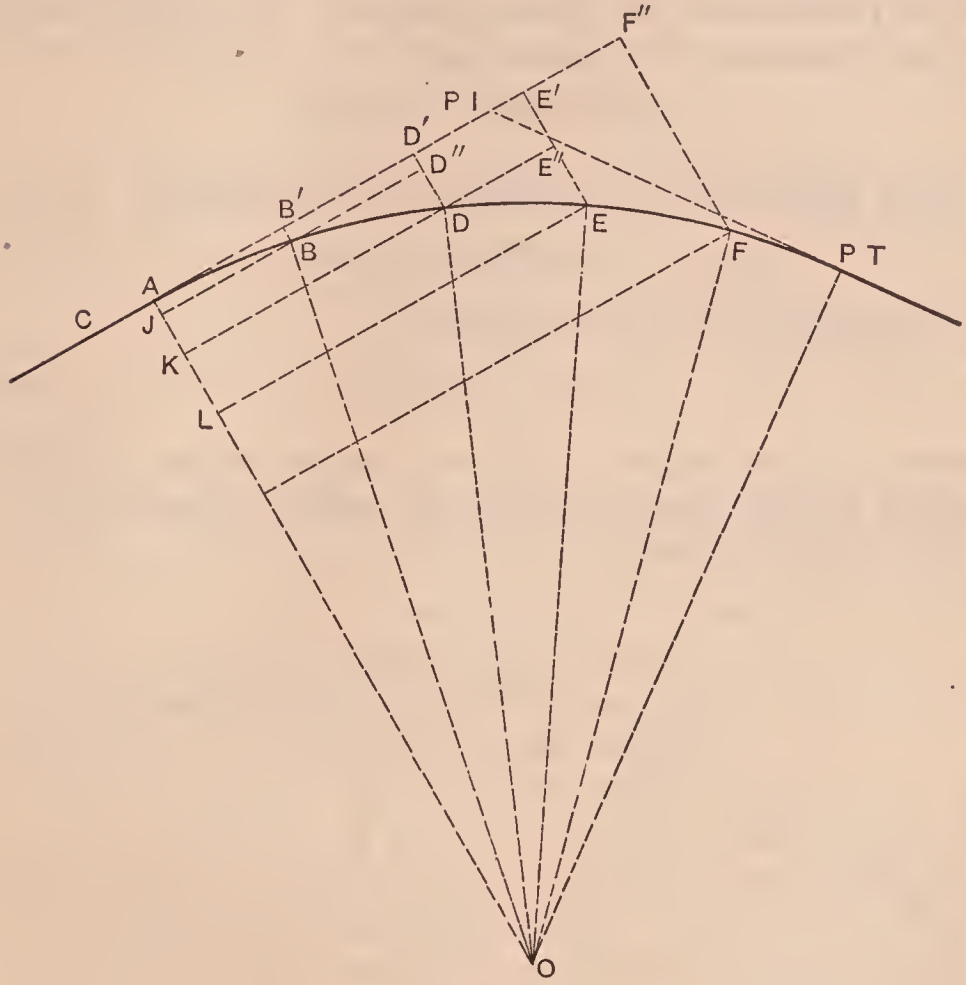


Fig. 9.

the tapemen may often be able to put in a flat curve accurately enough for grading the roadbed while the transitman is moving forward.

Another method of computing coördinates is often used. It has the advantage of giving the coördinates for each station directly and the disadvantage of using a large factor, R, as compared with summing the coördinates in the previous case and obtaining the increments, after the first, by simply pointing off two places in the natural functions. The equations are

$$\left.\begin{aligned} AB' &= R \sin (1 - n)\, D, \\ BB' &= R \operatorname{vers} (1 - n)\, D, \\ AD' &= R \sin (2 - n)\, D, \\ DD' &= R \operatorname{vers} (2 - n)\, D. \end{aligned}\right\} \qquad (20)$$

If the tangent point is at a full station, $n = 0$ and disappears from the formulas.

Offsets from tangents may be found approximately from the following:

Let n = the distance in stations from the tangent point to the point where the offset is desired. Then $\frac{1}{2}\,nD$ will be the angle between the tangent and the chord, and

$$\text{offset} = 100\,n \sin \tfrac{1}{2}\,nD.$$

Since small angles are proportional to their sines,

$$\sin nD = .01745\,nD^\circ \text{ (approx.)}.$$

Substituting, offset from tangent,

$$t = \tfrac{7}{8}\,n^2 D, \qquad (21)$$

where n is in stations and the corresponding deflection angle is so small that for its sine the angle in degrees into the sine of 1° can be used.

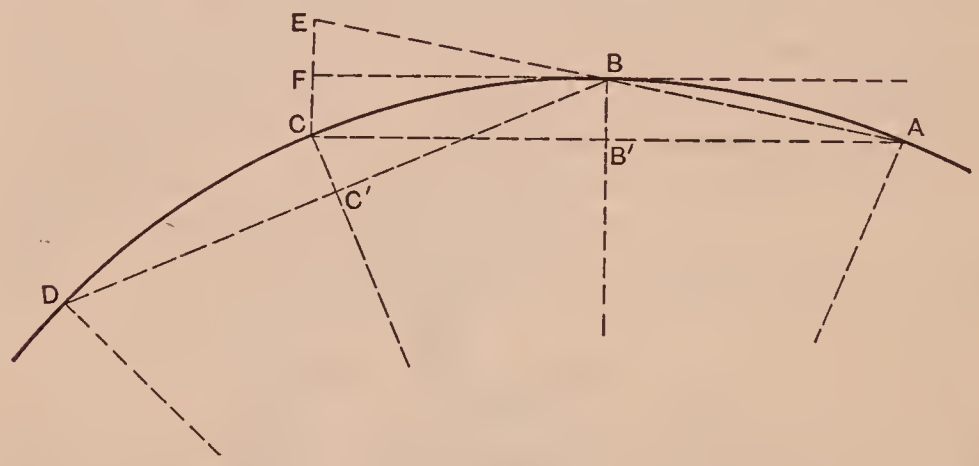


Fig. 10.

By writing a similar equation for a D' degree curve, and subtracting, distance between curves,

$$t' = \tfrac{7}{8}\,n^2\,(D - D'). \qquad (22)$$

This equation is useful for correcting trial curves or for determining the effect of changing D.

(*b*) by offsets from chords produced. — If in Fig. 10 the perpendicular CF be dropped from C to the tangent BF at the next full station B, the tangent offset, CF, or t, will subtend an angle CBF equal $D/2$.

$$\left.\begin{array}{ll} \text{Tangent offset,} & t = 100 \sin \tfrac{1}{2} D, \\ \text{or} & t = \tfrac{7}{8} D^\circ \text{ (approx.).} \end{array}\right\} \quad (23)$$

If CF be extended to meet the next 100-foot chord AB produced, at E, the chord offset CE, or o, will be twice the tangent offset.

$$\left.\begin{array}{ll} \text{Chord offset,} & o = 200 \sin \tfrac{1}{2} D, \\ \text{or} & o = \tfrac{7}{4} D^\circ \text{ (approx.).} \end{array}\right\} \quad (24)$$

In starting to lay out the curve from the tangent point, measure each way to a full station and lay off the tangent offsets by (17) or (21); prolong the chord through these two station points 100 feet forward and lay off the chord offset to locate the next station 100 feet from the front station through which the chord was drawn; extend the second chord and repeat.

This method is especially valuable in enabling the front tapeman to approximate the line on curves nearly as well as on tangents, thus saving time and checking the transitman. It is also valuable on construction in replacing the stakes approximately for checking the grading, etc., since it is usually possible to recover at least one of the chords and the curve may be extended either way from it.

(*c*) By deflections with two transits. The transits are set up at two different points on the curve, for example the PC and PT, and each set at the proper vernier reading for a required station; the intersection of the two lines of collimation will locate the station. The central angle of the curve between the two instruments should be great enough to give a fairly large intersection angle for accurate location. This method is sometimes used in locating points in a pond or other place where linear measurements are inconvenient.

These special methods are given to suggest expedients for convenience in field work. Others will suggest themselves to the thoughtful student as the result of experience.

The following approximate formulas in this connection should be memorized as greatly facilitating field-work.

$$\left.\begin{array}{l} t = \tfrac{7}{8} n^2 D, \\ t' = \tfrac{7}{8} n^2 (D - D'), \\ m = \tfrac{7}{32} n^2 D, \\ o = \tfrac{7}{4} D, \end{array}\right\} \quad (25)$$

where t = tangent offset; t' = offset between two curves; m = middle ordinate; and o = chord offset.

20. Obstacles to Alinement. — For transit work the deflection method is generally preferable to the offset method for passing obstacles as being quicker and more accurate.

The equilateral triangle, when available, involves no computations; the isosceles triangle involves computation for distance only, and this may be avoided by using deflection angles which will give simple ratios.

Thus any one of the following combinations of deflection angle and distance will give a projection forward of 100 feet and proportionally for other distances.

Deflection.	Distance.	Deflection.	Distance.
84° 15½′	500	56° 15′	90
82 49	400	51 19	80
80 24	300	44 25	70
75 31½	200	33 33½	60
70 32	150	24 37	55
60 00	100	00 00	50

The traverse with coördinates (example, §13) is perfectly general, but it requires computation. For small deflection angles from the main direction or axis of X the cosines may be taken as unity and the sines proportional to the angles, or since the ordinates or offsets are not needed for any intermediate point and the last ordinate is zero, the angles in minutes may be used instead of the sines. This gives the following:

Rule. — Call deflections to the right positive, and those to the left negative. Multiply the length of each course by the algebraic sum of the deflections up to that course and add the products. The main line is reached when the algebraic sum of these products is zero, and its direction is given when the algebraic sum of the deflections is zero.

Example. —

Station.	Deflection.		Algebraic		Product.	
	R	*L*	Sum		+	−
106	0° 50′		+50′	50 × 950	47 500	
115 + 50		1°10′	−20	20 × 250		5 000
118		0 30	−50	50 × 650		32 500
124 + 50					47 500	37 500

At sta. 124 + 50 the algebraic sum of the products is + 10000 and the direction of the line from sta. 118 to sta. 124 + 50 is 0° 50′ *L*.

If it is desired to reach the line in 5 stations, it would require a deflection of $-\frac{10\,000}{500} = 0°\ 20'$ to the left of the main line, or of 0° 50′ − 0° 20′ = 0° 30′ right from the last line or backsight, and at sta. 129 + 50 a deflection of 0° 20′ to the right, to continue the original line. Any other convenient distance could have been assumed and the corresponding deflections obtained.

On curves, if care is taken not to place the transit too close to a small obstacle, it can usually be passed by using chords which will give clear lines of sight for plus stations close enough together to carry the taping along the curve and then interpolate for the regular stations, or if this is not feasible, by setting a hub at the end of a long chord (computed by (5)), and then running the curve backward to the obstacle.

For larger obstacles one of the general methods already described can be applied to a long chord and the curve then run backward to the obstacle.

21. Alinement by Trial. — While the typical method of location is to fit the tangents to the ground and then connect them by suitable curves, as indicated in §12, the method becomes inconvenient when the intersection angles are large, as the points of intersection will come quite a distance from the located line, and the extended tangents may be on rough ground or obstructed, especially if in a timbered country.

Again, when the curves are long as compared with the tangents it is often better to fit them to the ground first rather than the tangents. For these reasons and because of the saving in time which can be effected, the method of running the curve forward from the *PC* without having the front tangent definitely located is much used. With this method special care is required to prevent mistakes, as no very precise checks on the curves are obtainable. The discrepancy between the computed and the magnetic bearing would show any serious mistake in the instrument work.

If the curve does not fit the ground it can be moved laterally by sliding the *PC* along the first tangent or by changing the degree of the curve as required. The proper point at which

to stop the curve can be determined by pacing off the tangent offset at the rear end of the tape and sighting over the stake at the front end and noting how the direction agrees with that of the required tangent. If it must be determined closer than to the nearest station, a plus station can be set temporarily and the corresponding tangent offset ($= \frac{7}{8} n^2 D$, or the chord $\times$ sine of deflection angle) laid off and the line tested before bringing up the instrument.

It is believed that this method is admissible up to 1000 feet for long curves, as any discrepancy found in the alinement can be readily taken care of by slightly shifting the positions of curves or tangents during construction.

PROBLEMS.

1. A line is measured along an even slope and found to be 670 feet long. One end is 50 feet higher than the other.

Find the true horizontal distance by (*a*) an exact method; (*b*) the approximate method.

2. A trial line measured through brush is found to be 890 feet long and to require an offset of 26 feet to pass through the required point.

Find the true distance by (*a*) an exact method; (*b*) the approximate method.

3. Given $M = 50$ ft. and $\Delta = 40° 25'$.
Required E, T, R and C.

4. Given $C = 300$ ft. and $\Delta = 10° 45'$.
Required E, T, D and M.

5. It is desired to put in the regular stations only on a 14° curve.

How much must the nominal 100-ft. chords be shortened to accomplish this, (*a*) if 50-ft. chords are to be used later; (*b*) if 100-ft. arcs are desired between stations?

6. A street railway curve of 90-ft. radius is swung in from its center.

Required the degree of the curve, (*a*) on the 100-ft. chord basis; (*b*) on the 100-ft. arc basis; (*c*) on the basis of using 10-ft. chords.

7. A 20° curve is to be run in at first with 100-ft. chords.

Required the deflection angles for putting the points on a curve to be run later with 25-ft. chords.

8. A curve is run with 5° deflections for 50-ft. chords.

Required the exact radius and the degree of the curve.

9. Find the coördinates, using the tangent as the axis of X, for locating 4 regular stations on a 5° curve with PC at sta. 180 + 20. Compare the ordinates with those found by the approximate method.

10. Find the coördinates from the long chord for laying out the regular stations of a 7° curve with *PC* at sta. 180 + 60 and *PT* at sta. 185 + 10.

11. Two tangents with bearings N. 60° 40′ E. and S. 80° 30′ E. intersect at sta. 180 + 4.0 metric with $\Delta = 39° 05'$ *R*. Computed bearing of first tangent [60° 29′].

Required complete transit notes for a 6° metric curve.

12. Given $\Delta = 36° 15'$ *L*; *PI* = sta. 260 + 6.0, metric; E = 10*m*. Bearings of tangents N. 70° 20′ W. and S. 73° 20′. Computed bearing of the first [70° 16′].

Required the complete transit notes.

13. Given $\Delta = 26° 35'$ *R*; *PI* = sta. 184 + 10.

Required the *PC* and *D* of a curve to pass through a point 10 feet to the right of sta. 182 on the tangent.

14. Given the following alinement: — sta. 176, *PC* 3° *R*; sta. 182, *PT*; sta. 185 + 50, *PC* 5° *R*; sta. 189, *PT*.

Required the *PCC*'s for substituting a 2° curve for the tangent between the 3° and 4° curves.

15. A 1° curve ends at sta. 181. On prolonging the tangent to sta. 190 it is found that 1° 30′ should be added to the curve to bring the tangent on the required ground.

Find the offset from the old tangent to sta. 182 + 50 and the distance and direction from the old sta. 190 to the new sta. 190 so as to set up on the new sta. 190 and backsight on the new 182 + 50 and run the new tangent without having to go back with the instrument.

16. A 4° curve is run from sta. 210 to sta. 216. On setting up at sta. 216 it is found that a 3° curve with the same *PC* would better fit the ground.

Find the distance and direction to lay off from the instrument to hub sta. 216 on the 3° curve and the vernier reading for sta. 217 after setting up on the new station and backsighting upon the *PC*.

CHAPTER II.

Construction.

22. Leveling. — As soon as the center line is located levels are run over it to determine the ground elevations. These should be taken at all stations and also at all breaks in grade in order to fully develop the ground surface. High-water marks and other drainage conditions should be carefully noted by the levelman, as also the classification or character of the soil, as these must be considered in fixing the height of the grade line upon the profile. Sea level datum should be used if readily obtainable.

Bench marks should be established at points about one-half mile apart for use on location if running a preliminary or for use on construction if running a location. They should usually be near grade points for convenience during construction but far enough away to avoid disturbance. Roots of trees, good masonry and large boulders make excellent *BM*'s for railroad work if properly marked and described; when not available, poorer ones have to be used.

With backsights and frontsights on turning points equal, or the sums between bench marks equal, and with sights limited to about 400 feet, there should be no difficulty in checking levels to less than 0.1 foot per mile on fairly rough ground, and there is no advantage worth spending money for in checking much closer than this on ordinary railroad location.

The readings on turning points only are placed in the *BS* and *FS* columns. The difference of the sums of these columns should check the last turning point or *HI*.

Thus,

$$24.86 - 11.37 = 13.49,$$

$$13.49 + 793.38 = 806.87, \text{ the last } HI.$$

The following is a convenient form for level notes.

LEVEL BOOK.

Sta.	*BS.*	*HI.*	*FS.*	Rod.	Elev.	Remarks.
BM.	11.59	804.97	.....	.....	793.38	Large stump,
⊙	12.61	817.41	0.17	.....	804.80	10 R. of 278.
272	.....	.....	.....	1.3	816.1	
273	.....	.....	.....	1.7	815.7	
274	.....	.....	.....	2.2	815.2	
+ 60	.....	.....	.....	4.1	813.3	
275	.....	.....	.....	5.7	811.7	
276	.....	.....	.....	5.4	812.0	
+ 40	.....	.....	.....	9.0	808.4	
+ 50	.....	.....	.....	12.3	805.1	Stream 3′ × 4′
+ 65	.....	.....	.....	8.4	809.0	Rock.
+ 80	.....	.....	.....	10.0	807.4	
⊙	0.66	806.87	11.20	.....	806.21	
+ 90	.....	.....	.....	1.9	805.0	
	24.86		11.37			

23. Projecting the Grade Line on the Profile.—The fundamental condition governing the fixing of the grade line is that the cuts shall balance the fills, as this gives minimum yardage to be moved and therefore cheapest construction for uniform material unless leads are thereby increased. An apparent balancing on the profile does not mean an actual balancing for the following reasons: —

1. The width of roadbed in cuts is made greater than on fills in order to give room for side ditches and sometimes the side slopes are different.

2. Earth shrinks on an average about 10 per cent from cut to fill, and rock swells some 60 to 75 per cent.

3. Steep transverse slopes increase the yardage for a given center height.

Using tables of level cuttings and making the allowances noted, even the novice can " balance cuts and fills " in time, but considerable experience is required to expeditiously lay on a grade line " by eye " which will satisfy with sufficient accuracy even the fundamental condition given above.

In fact it is usually unnecessary to accurately satisfy this condition, as one or more of the following factors enter into the question and good judgment and experience are required to strike a proper balance between them.

Excess excavation, or excess fill, may be required when the height of the grade line is modified by having to pass under or over an existing railroad or important highway, or when a location requiring light grading only would cost excessively for right of way.

Excess excavation is often required at summits in order to reduce the gradient to the required limit.

Embankment must be in excess over low, flat country, subject to overflow, where the grade line should be kept at least two feet above high water. First cost is often reduced by raising the grade line to reduce the cuts and using temporary trestles in place of the expensive portions of the fills.

When cuts are in solid rock or hardpan it is often more economical to raise the grade line and borrow cheaper material for the excess fills.

Deep cuts should be avoided in regions of heavy snow and whenever they would seriously increase the danger from landslides. While excess material may generally be used to advantage in widening out banks, it is best, as a rule, to keep the grade line fairly high, as fills tend to give cheaper maintenance on account of better drainage. A piece of fine thread is very useful in studying the projection of a grade line.

The grade line of this paragraph is usually called the subgrade; it defines the surface of the finished roadbed after grading. The ballast, ties and rails are placed on this roadbed, making the grade of the track a constant distance higher than the subgrade.

24. Volumes.— After fixing the grade line on the profile, the grade elevations or "grades" are computed for each station and, together with the ground elevations, entered in the cross-section book. The center cuts or fills are then made up as shown in the notes. It is desirable to have also a list of *BM*'s in the back of the book.

Tables of level cuttings could then be used to determine volumes if the ground were exactly level transversely. But as this rarely happens and since it is necessary to set slope

stakes at the edges of the cuts and fills to guide the contractor in grading, cross-sections are taken to determine the exact shape of each section. The cross-section notes are then available for accurately computing the volumes for which the contractor must be paid.

The simplest section is the two-level section in which the transverse slope is uniform, as in Fig. 11, making it necessary

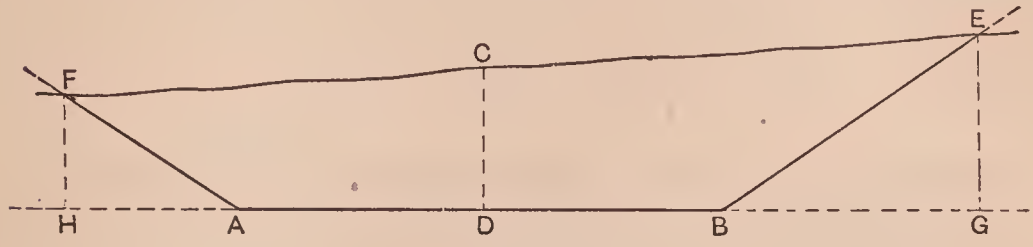


Fig. 11.

to determine the heights at the slope stakes, E and F, only, *i.e.*, at but two points. Since the center cut CD (or fill) is also known, however, this section is usually treated as a three-level section, just as if a break occurred at the center as in the true three-level section.

If the transverse slope breaks over or under the edges of the roadbed it is necessary to take readings at these points

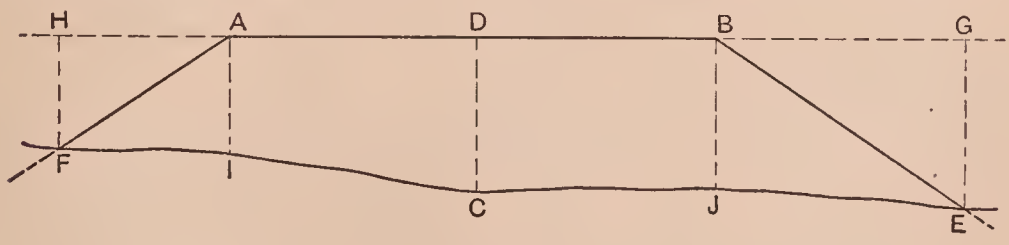


Fig. 12.

also, giving the five-level section shown in Fig. 12, *i.e.*, one in which five points are taken.

When the transverse slope is still more broken it is necessary to take readings at each break, giving an irregular section.

The volume between adjacent sections is often computed by simply multiplying the length between them by the average of the two end areas. This does not, however, give true volume, excepting in those rare cases when the earthwork solid is a prism or wedge. In the usual case the earthwork solid is a prismoid, which may be defined as a solid bounded by bases in parallel planes and whose sides are generated by

lines which move by proportional parts along the homologous sides of the figures determining the parallel ends. The sides may also be considered as generated by lines constantly parallel with the planes of the bases moving along the lines connecting homologous points of the ends, as directrices. Thus the sides are either planes or hyperbolic paraboloids, and the volume of such a solid may be computed by the following:

$$V = \frac{l}{6}(A + A' + 4\,A_m), \qquad (26)$$

where A and A' are the end areas and A_m the middle area.

From an inspection of this formula it is easily seen that the average end area method gives correct results only when A_m is an average of the two end areas. Since it would be a tedious operation to find A_m by the interpolation of all linear dimensions, it is usual to divide the more complex prismoids into triangular ones for the purpose of computation, especially in case the true volume is to be determined. The volume by averaging end areas is then found first, preferably by the use of tables (see Crandall's Tables for the Computation of Railway and Other Earthwork), and then a prismoidal correction is determined, also from tables. This correction depends upon the change in height and change in base between the two triangular ends.

25. Rules for Cross-sectioning. — From the definition of the prismoid it is evident that straight lines between any two homologous points of the parallel bases are elements of the surface, and this is the criterion which governs the field-work, and necessitates the following rules for taking cross-sections.

Take cross-sections at all regular stations and at plusses where necessary, so that straight lines connecting homologous points of adjacent sections shall lie wholly in the ground surface or equalize that surface. When the center line is on a curve, the lines connecting homologous points must be estimated at the same curvature in judging of their coincidence with the ground surface.

Take sufficient points in each cross-section so that straight lines joining them will lie in the surface or will equalize the surface.

The subdivision into triangular prismoids requires the same number of sides or points in adjacent cross-sections, so that, when a ridge or hollow runs out, its vanishing point in the more regular section must be taken and the section made up with the break for the solid on one side and without it for the solid on the other side of the section. The height of this vanishing point can be computed, but it is better to measure it in the field. An occasional extra break can often be treated best by itself, as adding or subtracting a computed amount to or from the regular quantity which would be obtained without it.

Cross-sections should also be taken wherever a grade point occurs on either edge of the roadbed (usually limited to the edges in cutting), and one is generally placed where a grade point occurs on the center line.

On light work and where the transverse slope is small, a section is sometimes taken where the center line comes to grade and the other two sections are omitted.

The plusses are preferably made 50 feet, multiples of 10 feet, or whole feet (except grade points), for convenience in computing grades in the field and quantities in the office. Breaks on a cross-section should be put at whole feet from the center when possible without a sacrifice of accuracy.

When the method by averaging end areas is used for computing the volumes of the prismoids, additional sections should be taken whenever the change in transverse slope or in area between adjacent sections is great, *i.e.*, when the change in height and width is great, giving a large prismoidal correction.

26. To take Cross-sections.—In taking cross-sections some engineers run a new line of levels and work from the *HI*'s. This is hardly worth while, especially if the *BM*'s have been used on the preliminary survey and rechecked on the location. It is desirable, however, to guard against local errors by checking between consecutive stations when convenient.

The usual process is to use the center cut or fill as a basis and from that determine cuts and fills at such other points as are necessary.

Thus (see form of Cross-section Book) the center cut at station 186 is 13.7. The rod is held on center and reads,

CROSS-SECTION

Sta.	Elev.	Grade.	Cut.	Fill.	Rod.
+50	853.8	869.6		15.8	
+16	866.3	869.5		3.2	
+08	869.3	869.5		0.2	
187	870.3	869.5	+0.3 0.8		
+90	872.2	869.5	2.7		
+50	879.6	869.3	10.3		20.3
186	882.9	869.2	13.7		20.4

say 6.7. The "grade rod" or reading of the rod if it could be held at grade would then be 20.4 (see rod column). That is, the line of sight is 20.4 above grade.

Then a reading is taken 10 feet out from center on the left. This is found to be 8.3, and subtracting this from the grade rod we have 12.1 for the cut at that point, which is noted as shown.

In this section it is next necessary to determine the position of the slope stake or where the side slope of the roadbed intersects the ground surface, as at E and F, Figs. 11 and 12. This point is found by trial.

In Figs. 11 and 12, EB and FA are the slopes whose inclination is expressed as the ratio of horizontal to vertical projection. Thus if $\frac{HA}{FH} = \frac{1\frac{1}{2}}{1}$, the slope is $1\frac{1}{2} : 1$, or $1\frac{1}{2}$ horizontal to 1 vertical.

Thus for a point to be on this slope, the distance from the edge of the roadbed must be equal to the cut or fill at the

BOOK.

Left.		Center.	Right.	
− 17.4 / 34.1	− 16.6 / 8	−15.8	− 14.0 / 8	− 12.6 / 26.9
− 9.7 / 22.5	− 6.9 / 8	− 3.2	00 / 10	
− 7.7 / 19.5	− 4.5 / 8	− 0.2	00 / 2	+ 1.1 / 10 + 1.4 / 12.1
− 1.0 / 9.5	− 0.3 / 8 00 / 5	+ 0.8	+ 2.4 / 10	+ 3.3 / 14.9
	00 / 10	+ 2.7	+ 4.3 / 10	+ 5.8 / 18.7
+ 6.3 / 19.4	+ 8.1 / 10	+ 10.3	+ 11.9 / 10	+ 15.8 / 33.7
+ 8.8 / 23.2	+ 12.1 / 10	+ 13.7	+ 15.0 / 10	+ 19.1 / 38.6

point multiplied by the slope ratio. The distances are always measured from the center line, which requires the half width of roadbed to be added in order to get the total distance.

Thus suppose the rod reads 11.1 at 20 feet from center. The cut at that point is then $20.4 - 11.1 = 9.3$; $1\frac{1}{2} \times 9.3 = 14.0$; $14.0 + 10 = 24.0$, which does not check with the measured distance, 20 feet. It shows, however, that the measured distance should be increased, so other points are tried until the correct one is found Thus the rod reading is 11.6 at 23.2 feet from center, *i.e.*, the cut is $20.4 - 11.6 = 8.8$; $1\frac{1}{2} \times 8.8 = 13.2$; $13.2 + 10 = 23.2$, which checks the distance out.

The slope stake is then marked C 8.8 and driven at 23.2 left of center. These stakes are usually faced towards and sloped away from the center line, so that they will not be confused with other stakes.

In like manner the rod is held over the edge of the roadbed

on the right (10 feet out) and a reading of 5.4 found, giving $20.4 - 5.4 = 15$ for the cut, which is noted as shown. The slope stake is then found as before.

Rapid cross-sectioning requires good judgment in the selection of points and ability to calculate quickly and mentally.

Apply the rule for distances out to each of the slope stakes given in the notes. The method of checking from one station to the next is indicated by the grade rod at 186 + 50. The rod reading at center is 10, giving $10 + 10.3 = 20.3$ for grade rod. Comparing the difference of grades with the difference of grade rods from same set-up for the adjacent sections, they are found to be the same, thus checking the work. Sometimes it will, of course, be necessary to carry levels over a turning point in order to apply this check.

The same principles as given above for cuts also apply to fills, the only difference being that the instrument may be under the grade line instead of over it.

A grade point occurs on center between 186 + 90 where the left side runs out of cut and 187 + 16 where the cut ends on the right. The fills do not begin, of course, at exactly these same points, but it is usual to consider them to do so, as it is not necessary to have the volumes of fills as accurately as those of the cuts.

Grade points along the center line are determined by moving the rod along the line until the reading equals the grade rod at the preceding station plus or minus the correction due to the gradient up to the point where the reading is taken, depending on whether the gradient is minus or plus. They may also be computed approximately from the notes by finding the rate of change of the surface. Thus in the notes given there is a grade point on center line between 187 and 187 + 08 whose distance from 187 is $\frac{0.8}{0.8 + 0.2} \times 8 = 6.4$ feet.

The same process serves to determine grade points at the edges of the roadbed except that the rod must be moved along a line parallel with the center line and distant from it by one-half the width of the roadbed.

27. Staking out Work. — The ordinary masonry structures like culverts, bridge abutments, trestle piers, etc., usually consist of two parts, the foundation and the neat work. For convenience, or to increase the bearing area, the founda-

tion is usually larger and the excavation pit is dug the full size or more for the foundation. When the foundation is built the face lines for the neat work are drawn and the masons or carpenters work to them for the portions of the structure in sight. If the face walls are battered, allowance must be made at the bases so that the tops will be in their proper positions. If the structure does not extend up to the roadbed, the exact distance below the latter is not required in laying out the work unless it is limited by the roadbed side slopes. In this case the relation between distance out from center and depth below grade is determined as for cross-sectioning.

For bridge masonry, trestle piers, etc., the office plans will show the height with reference to tops of ties or subgrade as the case may be, and this height must be worked to accurately from foundation up or proper allowances made in locating the neat work lines on the ground.

Masons use a string in laying stone to a face line, stretching it first between the face line stakes given by the engineer and afterwards between corner stones set with a batter board at the proper slope.

When it is necessary to extend a curve across a bridge its axis should be a chord which will leave the same lateral clearance at the center as at the ends and the masonry staked with reference to this the same as if the track were straight. The bridge may of course require widening on account of the curve.

Great care is required in laying out structures to avoid mistakes and inaccuracies. If plans are not furnished, a sketch plan should be made in the notebook showing all stakes and dimensions or other data used. If plans are furnished, the sketch should also be made showing the field data. These field data should be reviewed at night as a further precaution against mistakes.

28. To stake out a Bridge Abutment. — The general plan and section are shown in Fig. 13. The subgrade elevation at the left abutment is 196.22, and the bottom of the neat work of the abutment is 176.22. The face batter under the coping is one inch to a foot.

Set up on the center line, AB, of the roadbed and point C for the base of the neat work face and D for the foundation.

Set up at C, turn 90° and set E and F for the corners of the wing walls; extend this face line say 20 feet each way and put in solid reference hubs to preserve the face line. Set up at E and lay off 30° for the face of the wing wall. If staking from a general plan, the end of the wing G can be found by making its distance from $AB = 1.5$ times depth

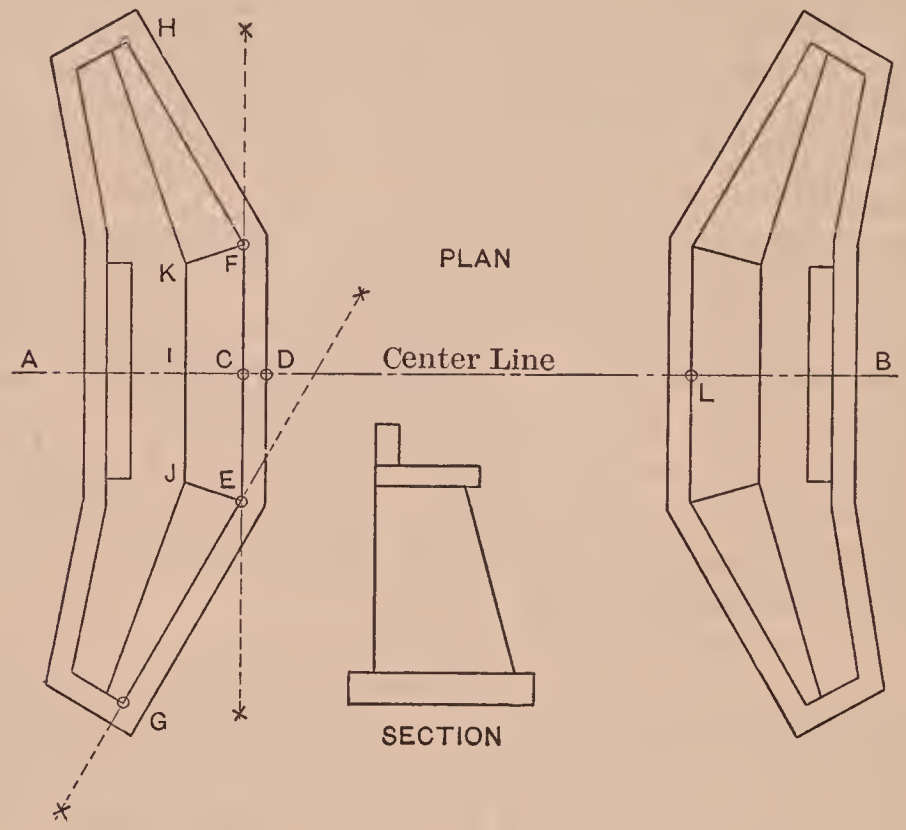


Fig. 13.

below grade, plus half width of roadbed. It is customary to make the height of the end of the wing 3 or 4 feet and allow the fill to slope slightly around the end. If staking from a plan, the length would be given and could be laid off directly. Reference hubs can be placed on EG produced at such distances as not to be disturbed by the excavation.

Similarly the wing FH can be staked and referenced. Having these face lines the foundation lines can be laid off from them without an instrument, or they can be laid off independently from D.

The excavation should be carried below frost if on soil, while other considerations such as danger of undercutting or poor material may require it to be carried much deeper.

Levels are run from the nearest *BM* and the *HI*, say 179.84, found for the last set-up. Then 179.84 − 176.22 = 3.62, the rod reading for top of foundation masonry. Drive substantial stakes, one near the center and one near each end, until the rod reads 3.62 when held on the top. These stakes should be near enough so that the masons' level board will reach the masonry.

The face batter will be carried up the quoins, *JE* and *KF*, and its value should be computed so as to give the 1 inch per foot on *IC*.

For coursed masonry the height should be checked from time to time with the drawings to insure the bridge seat's coming at the correct height.

The distance *CI* in inches = height of battered face in feet; the clear span under the coping will be greater than that at the base by twice *CI*.

If the span is long, or there are several spans between the abutments, it is necessary that the tape agree with the bridge company's standard for the structure to fit the masonry.

29. Referencing a Line. — During construction practically all the stakes and hubs on the center line are dug out or covered. This requires that the line be referenced for ease of recovery in running track centers or even for giving line and grade as may be necessary from time to time during the progress of the work.

If the *PI*'s are outside the slope stakes and accessible they are excellent points to preserve by substantial hubs. When these are not suitably situated, the *PC*'s and *PT*'s may be referenced to hubs or other objects outside the slope stakes. This may be done by measuring a distance each way at right angles to the line, or by measuring two distances on the same side. If there is a deep cut or high fill to be made at the point, the remeasurements will be inconvenient and the exact recovery difficult.

On this account, many prefer intersections obtained by setting the transit over the center hub, turning 45° or other convenient angle from the line so that four hubs may be set well outside of the slope stakes with angles of 90° between adjacent ones. This gives two intersecting lines at right angles, which should give a good recovery even on rough ground but is not convenient if the view is cut off by the con-

struction of a high fill. Approximate distances should be measured to aid in finding the hubs.

30. Track-laying and Monumenting. — In recovering the line for track centers after the grading is completed, some discrepancies occur in relocating the referenced points; these added to the inaccuracies of taping on rough ground and possibly to the inaccuracy of the instrumental work, usually necessitate some readjustment of alinement. Some shifting may also be necessary in order to fit the roadbed and structures. With light grading and short curves the tangents should be located and the *PI*'s used in running the curves as in §12. If the *PI*'s are inconvenient of access, due to heavy grading or long curves, the curves can best be run by trial from the imperfectly recovered *PC*'s and then corrected by sliding the *PC*'s along the tangents as in §15, converting any slight discrepancy in Δ into distance at the *PT*. On very long curves a slight compounding may sometimes be required to follow the roadbed. This is not especially objectionable if the break in curvature is small.

If on location care was not taken to level up the tape so as to bring the stations 100 feet apart, the tape should be held correspondingly short in running track centers so that the stations will coincide as nearly as may be with their old positions; otherwise the gradients, property lines and alinement will all be disturbed and confusion will result. After the line is in operation an entire resurvey can be made if desired and all these changes taken into account.

The stakes should be $1\frac{1}{2}$ or 2 inches square and well driven; they should be pointed with tacks on curves.

The grade stakes are driven at the side with their tops at the height of the top of the rail, allowing for vertical curves at all breaks in gradients.

The tangent points, points of change of curvature, summits and, if transition curves are used, the ends of the transition curves, should be marked by permanent monuments to guide the track foreman in maintaining alinement and for use in realining the track with an instrument. These should be set on the side at a convenient but uniform distance from center. Stones or pieces of old rail, from 3 to 5 feet long according to soil, and centered, are much used.

It is the custom of many roads to monument also their

right of way and other property lines for convenience and protection.

Permanent *BM*'s should also be established, but this can usually be done on masonry structures or other permanent well-defined objects without having to place them specially for the purpose.

PROBLEMS.

1. Find the distance out to a slope stake 20.3 feet above grade for a 20-ft. roadbed and a $1\frac{1}{4}$ to 1 side slope.

2. Find the width between trusses for a clearance of 7 feet from center of track for a bridge 90 feet long on a 3° curve.

3. In separating grades in a city the subgrade is raised 13 feet. Find the height of retaining wall required to hold the slopes for a double track roadbed 28 feet wide on a 40-ft. right of way.

4. A steel-plate girder trestle as shown in Fig. 14 is to be built on a 6° curve. Draw a sketch and show on it the data required for staking out the centers of the post piers.

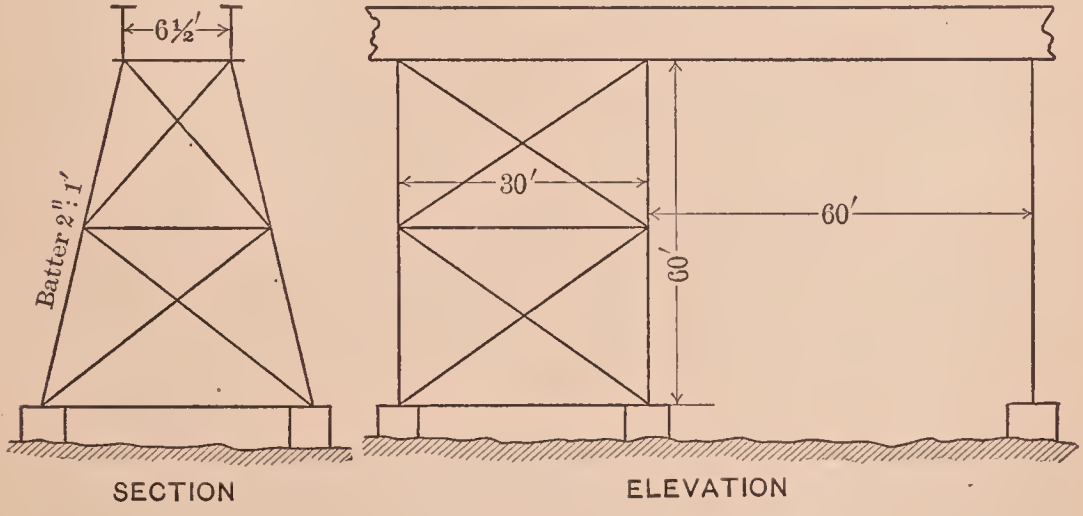


Fig. 14.

5. An overhead highway bridge 20 feet wide is on an 8 per cent gradient and makes an angle of 50° with the railroad center line.

(*a*) Find the clear span for a clearance of 14 feet for the railroad.

(*b*) Find the elevations for the tops of the piles of the pile abutments, using a 10 × 10 inch cap, for a clearance of 22 feet above center of track. Elevation of track 462.75.

6. Find the length of 12-inch culvert pipe required under a 20-ft. fill for a 16-ft. roadbed with $1\frac{1}{2}$ to 1 side slopes and a 4 per cent transverse surface slope.

7. Find the increased distance required on the inside, Prob. 2, to give the same car clearance if the outer rail is elevated 3 inches; top of car body 14 feet above rail.

8. The preliminary examination of a cut indicates that rock will be found at a depth of 12 feet. Find the distance out to a slope stake having a cut of 30 feet for a 27-ft. roadbed; rock side slope $\frac{1}{4}$ to 1; earth slope $1\frac{1}{2}$ to 1.

9. If 14 feet is the minimum safe clearance in a rock cut on a tangent (vertical sides), find that required for a 10° curve, outside and inside, for an elevation of outer rail of 7 inches, with a car body 14 feet high and 70 feet long pivoted to the trucks at points 48 feet apart.

10. The abutments of a bridge make an angle of 60° with the axis and are 20 and 25 feet high, respectively.

Find the distance between face lines at the base for a clear span of 110 feet under the coping and a face batter of one inch per foot.

CHAPTER III.

Reconnoissance.

31. Survey for Location.— A survey for location generally consists of three more or less complete surveys: —

a. The Reconnoissance. — A rough, rapid survey, or examination, of the entire belt of country within which there is any probability of finding a practicable route.

b. The Preliminary Survey. — An instrumental survey or traverse of the line, following closely the route picked out on reconnoissance and serving, with its topography, as a basis for —

c. The Location. — A transit and tape survey, in which the curves are run in and the center line is staked on the ground.

These surveys, of course, generally increase in accuracy from the first to the third and each should vary in character with the local conditions.

The problem may come to the Locating Engineer in its very inception as a project to build a line between two widely separated points, when all the broad questions of traffic, etc., indicated in the Introduction, must be considered in first selecting general routes and finally narrowing down to the located line.

Usually, however, quite definite instructions are given as to general route, maximum gradient, degree of curve, value of distance, curvature, rise and fall, etc., by the higher officials, who have themselves considered the larger questions involved. The problem is then to find the best line obtainable which will satisfy the requirements of the instructions. If no such line can be found, the Locating Engineer should report that fact together with suggestions as to the best procedure to overcome the difficulty.

32. The Reconnoissance. — The great importance of the location of a line was shown in the Introduction, and it is now easily appreciated that the reconnoissance is its most important part, as it usually fixes the general route. That is, better

lines are rarely found after the completion of the reconnoissance, unless by competitors, since the selection of one route eliminates all others from consideration.

Sometimes two routes will be found so nearly equal in desirability that a more accurate survey is necessary in order to decide between them, but at other times a single route can be picked out on reconnoissance.

Hence the importance of employing the best possible talent for the survey for location and especially for the reconnoissance cannot be too strongly emphasized, as many of the worst errors in railroad location are made on the reconnoissance.

It is also evident that the reconnoissance cannot be fully discussed without a knowledge of construction and operation, so attention will now be confined to the instruments and methods used and the more obvious effects of topography in placing the line.

33. Types of Lines. — Flat country, with natural slopes not exceeding the desired maximum gradient, presents practically no problem in location, the only thing necessary being to connect the controlling points, *i.e.*, stream crossings, towns, etc., by straight lines. It need hardly be said that such conditions are rare.

In country approaching the above, especial care must be taken to avoid long, shallow cuts on account of lack of drainage. Such country is considered somewhat difficult largely because it is deceptive; the apparently smooth, flat slopes giving, when plotted, steep gradients or heavy work which cannot be reduced without large lateral deviations on account of the flat transverse slopes.

About the simplest of the real location problems is presented where a road ascends the side of a valley for the purpose of reaching a town or a low saddle where it can pass into the adjoining watershed. Rough country may, of course, make the details of the location difficult, and the problem itself requires careful study if the line is to be run on ruling gradient, and especially if development is necessary in order to keep below the desired maximum gradient.

A strictly valley line, in which the road runs along a stream and the advantages are alternately on one side and then the other, usually presents a more difficult problem. Both banks must be carefully examined in this case and detailed com-

parisons made between the most promising schemes, taking into account not only the actual costs of the right of way, cuts, fills and bridging (not only of main stream but of tributaries), but also the less tangible points, such as liability to landslides or flooding, advantages of reducing gradient, curvature, distance and rise and fall, and of increasing traffic.

A ridge line is apt to require heavy work in order to be kept fairly straight, but on the whole is probably less difficult than a valley line.

The most difficult general problem will be presented when the line cuts directly across the main drainage, since this gives minimum distances and maximum differences of elevation. High stream crossings and low saddles must be sought, but are rarely sufficient in themselves to give satisfactory gradients, and heavy summit cuts or even tunnels and "development," or purposely lengthening the line by curves, switchbacks or loops, are more frequently necessary in this case than in the others.

In cutting diagonally across the main drainage, advantage may be taken of the valleys of secondary streams on one side, but long detours are often necessary on the other, though tertiary streams are sometimes of use, especially if fairly large.

Few lines are entirely of one type and most will present a mixture of several different types. It is evident, however, that the difficulty of selecting the general route on reconnoissance will vary with the type of line as discussed above, while those attendant upon the detailed adjustment of the line to the ground will vary with the roughness or brokenness of the country.

Hence the engineer on reconnoissance should study every detail of the drainage, upon which the type of line has been shown to depend, and not allow the ruggedness of the topography, especially if in short stretches only, to exert undue influence in placing the line.

34. Office Preparation. — The first step in preparing for the reconnoissance is to collect all available maps or other records. Accurate topographical maps to large scale are practically never available and perhaps the most useful maps obtainable are the small scale topographical sheets published

by the United States Geological Survey, which are now available over large sections of the country. The general routes can be selected and approximately plotted on these, even in fairly difficult country, thus materially reducing the labor of the reconnoissance.

In laying out general routes the approximate gradients between controlling points are found from the differences of elevation, allowing for cuts and fills, if any, and the approximate distances as scaled from the map. If the gradient thus determined does not exceed the adopted maximum, the line is then laid out by setting a pair of dividers at the proper distance to give on that gradient a change of elevation equal to the contour interval and then stepping along the line from one contour to the next. Care must be taken to keep as nearly as possible to the proper position for the located line by jumping straight across narrow ravines and through narrow spurs instead of following the contours exactly. The distance stepped off in this way should be more accurate than that scaled directly, and it may be necessary to revise the grade, when allowance may also be made for curvature if necessary. The method of following up this line on the ground will be practically the same as establishing a line, which process will be described later.

Maps without contours, such as Land Office, County or State Maps, will be available for large territories which have not yet been covered by topographic surveys, but these are useful principally for the determination of distances. These outline or skeleton maps will usually show rivers and streams, however, and an experienced man will derive much information from a study of the drainage systems alone.

35. Field Methods.—When no maps can be obtained and also when a general route as plotted is to be followed up in the field, means of determining distances, differences of elevation and direction are necessary. Since the reconnoissance is only a rough survey, hand or pocket instruments suffice for this work, though very extended exploratory surveys in new countries require the use of a much more elaborate outfit, including astronomical instruments, mercurial barometers, etc. Such surveys will not be treated here. See Godwin's Railroad Engineer's Field Book.

Direction is determined by the azimuth compass. Many

of these are not very sensitive, and it is best to get the bearing of a line by taking the average of two readings obtained by swinging up to it from each direction.

Approximate distances are obtained by scaling from a map, by timing, by the odometer or pedometer, by pacing and by estimation.

The method of scaling is good for long distances when points can be located from the map with fair accuracy and the map itself is correct. No map should be accepted as correct, however, until thoroughly tested. The errors occasionally found on the Geological Survey sheets are usually such as the omission of roads, streams, etc., which do not affect distances but may confuse one as to his location. Sometimes the elevations are incorrect, necessitating a revision or even abandonment of a general route. The Locating Engineer should be constantly on the lookout for these errors and apply every possible check as he proceeds.

Distances may be obtained within the necessary limits by timing, whether the method of travel be by wagon, on horseback or on foot. This is especially true where checks can be obtained at long intervals, say such as may be covered in a day or half a day, by the use of a map, as the intermediate points may then be checked by interpolation.

The odometer may be used for reconnoissance from a wagon and the pedometer when on foot. Reconnoissance from a wagon is not desirable unless the country be fairly easy, and the line obviously should lie near the road. Even the most energetic engineer, with a firm determination to investigate on foot all points requiring it, will often miss seeing what he should see, due to the use of the wagon.

The engineer on horseback is more free to follow the lines unless high fences and private rights prevent. Longer distances can of course be covered and with less fatigue than on foot. Very difficult country must be investigated on foot and much that is not difficult is covered in this way. Pacing must also be resorted to in obtaining distances, unless one is sufficiently experienced to estimate them closely. Timing and the pedometer are useless where there is much doubling back and forth.

It should be carefully remembered that the more difficult the problem and the narrower its limits, the greater the

accuracy required. Lines are even roughly taped in some instances.

The hand level is somewhat used for the determination of elevations, especially at critical points and in open country. The aneroid barometer is, however, more extensively employed, as it is much more rapid, can be used in wooded country and is accurate enough for most reconnoissance purposes if intelligently used.

36. The Barometer. — This is an instrument for measuring the pressure of the atmosphere. It is done in the case of the mercurial barometer by measuring the height of the column of mercury which the atmosphere will support. Pure mercury is put into a glass tube about three feet long and one-quarter inch in diameter and carefully boiled to remove all air and moisture.* It is then inverted into the cistern of mercury, when the column lowers until the pressure balances that of the atmosphere, leaving a nearly perfect vacuum about one-half foot long at the top of the tube. For the standard mountain barometer, this tube is supported in a brass case which extends to the top of the cistern. The top of the cistern is a glass cylinder; the lower portion is of wood and the bottom a chamois bag. This extension is contained in a brass cylinder which is attached to the upper one by screws through outside flanges, leaving the glass portion of the cylinder exposed. An ivory point projects into the cistern from above and an adjusting screw from below supports the chamois bag.

For a reading, the top of the mercury in the cistern is raised by the adjusting screw acting upon the chamois bottom until the light just disappears under the ivory point without causing a dimple, when the vernier tube is moved so that the plane of its bottom is tangent to the spherical top of the column as seen through two opposite slits in the brass case.

The vernier reads to 0.002 of an inch, and the scale is ad-

* A field method of replacing a broken tube is to fill with mercury, then invert, holding a piece of chamois over the end of the tube, and allow a small portion of the mercury to run out; the rarefied bubble remaining is then moved along the tube and the small air bubbles gathered up. The tube is refilled and the process repeated several times. A tube carefully filled in this way will give readings but a few hundredths lower than a boiled tube.

justable with reference to the ivory point for its zero, allowing for the capillary depression of the tube.

The attached thermometer is read first, before it has been heated by the observer. The reading is necessary to correct for the expansion of the mercury and scale.

In moving the mercurial barometer, the adjusting screw at the bottom should be turned up until the cistern and finally the tube are filled, taking care not to press hard enough to force a leak in the cistern. When the tube is apparently filled, a gentle inclination with the ear near it will indicate by a metallic click if it is safe to invert without further turning the screw. If not almost filled, there is danger of the tube's breaking with the blow from the mercury; if pressed too hard, the mercury will leak at the cistern.

After inverting, the screw should be backed off about one-quarter turn to allow for the expansion with rise of temperature.

The aneroid barometer consists essentially of a box from which nearly all the air has been exhausted and which has its sides held apart by a spring which balances the atmospheric pressure and allows the box to expand or contract with change of pressure. This small motion of the sides is multiplied and communicated to an index moving over a graduated scale.

The aneroid cannot be a standard barometer, as there is no method of graduating its scale except by comparison with a mercurial barometer. It is graduated in inches of mercury, and has an adjusting screw for shifting the zero from time to time, as it moves due to the wear or disturbance of the delicate mechanism of its parts.

It can thus never be depended upon for actual pressures except when frequently compared with a standard mercurial barometer. It can, however, be depended upon to give correct differences of pressure during short intervals of time if well made and carefully handled.

Most aneroids also have an altitude scale which may be used for hypsometric work. It is graduated in feet to correspond with the pressure scale for a standard air temperature and its zero corresponds with standard pressure at sea level or other assumed datum.

The instrument should be compensated for temperature which is said to be done by cutting out a portion of the side of one of the brass levers and replacing it with steel; differential

expansion bending the lever to counteract the expansion of the other parts.

For hints in purchasing and testing an aneroid, reference is made to Trans. Am. Soc. C. E., Vol. I, p. 277 (The Aneroid Barometer and its Use in Estimating Altitudes, by Gen. T. G. Ellis.)

In using the aneroid, bring the face to a horizontal position and tap the case gently before reading. It should be protected from jarring and sudden changes of temperature, hence it should be kept in the case and neither carried too close to the body nor exposed to direct sunlight. The index adjustment should not be disturbed too frequently, it being better to apply a small index correction.

37. Barometric Formulas.—The relation between atmospheric pressure and altitude is a logarithmic one on account of the decrease of density with altitude. Professor Church (Mechanics, p. 620) derives the following expression:

$$h = \frac{p_0}{\gamma_0} \cdot \frac{T_n}{T_0} \cdot \log_\epsilon \left(\frac{p_n}{p_m}\right), \qquad (27)$$

where h is the difference in elevation; p_0, the standard pressure, 14.701 pounds per square inch; γ_0, the corresponding weight, 0.08076 pounds per cubic foot, for atmospheric air; T_n, the absolute temperature and T_0, the absolute temperature of the freezing point, = 273° C.; p_n and p_m, the observed pressures; and $\log_\epsilon$, the Naperian log.

In applying this to barometric leveling there should be corrections for the variation of gravity with altitude in its effect on the weight of the air and for the difference in density due to humidity. Both of these corrections are small and may be omitted for reconnoissance work. Before substituting the ratio of barometric readings (B_n/B_m) for that of pressures (p_n/p_m) all instrumental corrections must have been made.

Mercurial barometer corrections.

1. Reduction to standard temperatures, 32° F. for the mercury and 62° F. for the brass scale.

2. Correction for the difference in the force of gravity acting upon the mercury in different latitudes.

3. Correction for the difference in the force of gravity due to elevation above sea level.
4. Correction for capillarity of the tube.

The first correction is taken from Table VIII, which was computed by the following formula:

$$C = -B \frac{m(t - 32°) - l(t - 62°)}{1 + m(t - 32°)}, \tag{28}$$

where C is the correction; B, the barometric reading in inches of mercury; m, the coefficient of expansion of mercury, = 0.000 101; t, the temperature as read from the attached thermometer; and l, the coefficient of expansion of the brass scale, = 0.000 010 2.

This is therefore an instrumental correction for the purpose of eliminating the effect of the expansion of the mercury and the scale from the readings, and must be applied before the values are used for any purpose. The other corrections are taken care of in the formula for difference of elevation, (29).

Aneroid barometer corrections.

1. Correction for scale errors.
2. Correction for temperature.
3. Correction for index error.

These should be determined by comparison with a standard mercurial barometer at different temperatures, the mercurial readings having been reduced to 32° F. at sea level in latitude 45°.

The best aneroids have no appreciable scale errors, while they are compensated for the temperature of the instrument. For an unknown instrument this should be proven by test, not taken for granted.

If the same instrument is used for both readings the correction for index error, unless large, may be omitted.

There are many formulas in use for the determination of heights, which vary in form and somewhat in the constants used. The following is given in the Smithsonian Meteorological Tables published in 1893 as the result of a study of the best available constants:

$$Z \text{ (feet)} = 62583.6 \log \frac{B_0}{B} \left[\begin{array}{l} [1 + 0.002039\ (t - 50°)] \\ (1 + \beta) \\ (1 + 0.002662 \cos 2\phi)\ (1 + 0.00239) \\ \left(1 + \dfrac{Z + 2\,h_0}{R}\right) \end{array} \right] \tag{29}$$

in which 62583.6 (log B_0 − log B) is an approximate value of Z and the factors in the parentheses are correction factors depending respectively on the air temperature, the humidity, the variation of gravity with latitude, the variation of gravity with altitude in its effect on the weight of mercury in the barometer, and the variation of gravity with altitude in its effect on the weight of air.

For convenience in tabulating, log B_0 − log B is replaced by $\log \dfrac{29.9}{B} - \log \dfrac{29.9}{B_0}$.

Table IX contains values of $62583.6 \log \dfrac{29.9}{B}$, with B as argument.

The table is entered with the corrected barometer reading for each station and the difference of tabular quantities taken for the approximate difference in elevation. The temperature correction can be taken from Table X, or computed mentally by adding $\frac{1}{5}$ of 1 per cent for each 1° above 50° F., or subtracting $\frac{1}{5}$ of 1 per cent for each 1° below 50° F., using the average temperature for the two stations.

The other corrections are small, and reference is made to the Smithsonian Tables for work in which they are needed.

Professor Airy's Table, which was made for graduating the altitude scale of aneroids, may be of value in making comparisons. It is expressed by the following (Gen. Ellis, Trans. Am. Soc. C. E., Vol. I, p. 301):

$$Z = 62759 \log \frac{B_0}{B} \left(1 + \frac{t + t' - 100}{1000}\right) \tag{30}$$

where t and t' are the observed air temperatures at the two stations. He used 31 for B_0, or for the zero of his altitude scale, which is 982 feet lower than the zero of Table IX or 985 by the above formula, for a temperature of 50° F.

Formula (30) gives the following:

Inch Scale.	Alt. Scale.	Inch Scale.	Alt. Scale.
31	0	28	2774
30.5	443	27.5	3265
30	894	27	3765
29.5	1352	26.5	4275
29	1818	26	4794
28.5	2291	25.5	5323

38. Barometric Field Work. — The above formulas for difference of elevation suppose simultaneous readings at points not far apart horizontally. However, on extensive government barometric surveys west of the 100th meridian, where sets of simultaneous mercurial barometer field readings were compared with the daily readings at the different meteorological stations, those nearest in altitude gave better results for elevations than those nearest in distance. This was in a section where the differences in elevation were great.

To secure simultaneous readings, two barometers are required, one at headquarters or at some fixed point and the other in the field. If the one at headquarters is read at the whole and half hours, intermediate readings can be interpolated for comparison with the non-simultaneous field readings.

The index error of the field aneroid should be determined each day before and after the field work and the correction applied before making comparison with the office record. Any slight change between morning and evening should be distributed proportionately to the time. Any large change, indicating a disturbance of the instrument, should exclude the readings unless the time of the change can be located and a constant value established for a sufficient time before and after the disturbance to warrant working up the results.

Instead of determining the index error at headquarters, the morning and evening readings may be taken at a point

of known elevation and a table of probable readings for this point made up by applying the constant difference to the office readings, this difference including index error and difference of altitude.

An accurate barograph will give a continuous office record without the expense of an observer.

With a single aneroid, satisfactory results can be secured for short intervals of time only, on account of the erratic changes in the pressure of the atmosphere. For short intervals of time the rate of change may be considered constant, and approximate results obtained by reading at one point, then at a second, and again at the first, and comparing a reading interpolated for time for the first with the reading at the second for finding difference of elevation.

Another method is to determine the rate of change of the barometer at as many different times as is convenient during the day, by taking two readings at the same point some 15 to 30 minutes apart. These rates are then extended both backward and forward over about one-half the intervening time.

The altitude scale should not be used except for small differences of elevation unless it has been compared with the Airy or other standard formula to see if the readings correspond with the pressure. If the altitude scale is movable, it should be set in its normal position, otherwise it will not give correct differences of elevation.

An accurate altitude scale takes the place of Table IX, and requires only the temperature correction, which by the Airy formula is 0.2 per cent per degree above or below 50° F., + if above, − if below, and it can be computed and applied mentally.

A movable altitude scale allows of setting the zero with the pointer at a given station, for running gradients or for reading a certain small number of feet above or below the station; it is not intended for accurate leveling.

If the altitude scale is used in the field, it will usually be less work to apply the index correction (with the sign changed) to the office barometer and then enter the corresponding altitude scale readings for comparison with the field readings.

Barometric levels should always be accepted with a reservation unless they have been carefully checked by duplicate readings at important points and under varying conditions. Usually the results will be good, but occasionally large discrepancies appear which cannot be accounted for by lack of care or by poor weather conditions which were apparent at the time.

Even the best aneroids will sometimes "drag," *i.e.*, not come to the proper reading at once after a considerable change of elevation. This necessitates a short delay at stations until the reading becomes constant.

EXAMPLE. — The following office readings were taken with a Green standard mountain barometer and a K. & E. aneroid.

Time.	Attached thermometer.	Barometer.		Aneroid.	
		Observed.	Reduced (T. VIII).	Observed.	Correction.
7 A.M.	60° F.	29.642	29.558	29.482	+.076
8	62	.640	.550		
9	65	.635	.537		
10	70	.632	.521		
11	72	.624	.508		
12	76	.623	.497		
1 P.M.	77	.624	.495	29.417	+.078

At 10.15 the aneroid read 29.104 with $t = 75°$ in the field. Required the difference in elevation.

Table IX, B	= 29.518	350
B_1	= 29.181	662
Approx. Z		312
Table X, t	= 73°	+ 15
Difference of elevation		= 327

The altitude scale would have given 1720 for the field reading and 1407 for the office reading (29.517 − .077 = 29.440), giving a difference of 313

0.1% of 46° = .046; .046 × 313 = 14

Difference of elevation = 327

39. The Preliminary Survey. — After the Locating Engineer has presented his report on the reconnoissance, giving the results briefly and including a rough estimate of the cost of construction, it may be decided to run a preliminary survey. This will consist of a topographical survey of a narrow belt of country following the general route or routes picked out on reconnoissance and within which it is estimated that the final located line will lie.

If necessary to make such a survey over more than one route in order to compare them, it is desirable that the line traversed approach very closely the trial location so that fairly accurate estimates of cost of construction may be made. Sometimes very little or no topography will be taken in this case and courses will be short in making swings so as to approximate very closely the trial location.

Some engineers attempt to pick out courses which can be used for tangents in the final location even when but one route is to be surveyed and then take topography only at critical points. Others simply survey a line close enough to the single general route so that the topography will surely cover the located line.

Even in this latter case practice differs in respect to the accuracy required. Some make a very accurate survey and take detailed topography. The line is then plotted by coördinates and the topography carefully mapped. The location is projected, its coördinates determined and transit notes worked up which are then run in by the transitman in the field. Frequent ties to the preliminary are furnished him for checking his work.

Other engineers do not attempt to make the preliminary especially accurate but use the projected or paper location simply as a guide for the final location, which they themselves make in the field. This keeps them constantly studying the country, and it is believed to be the better method. This question is more fully discussed in the books on railroad location, and is mentioned here only for the purpose of indicating the differences in character and purpose of preliminary surveys.

In any case a traverse should be run, using the general field methods given in the Introduction. This is followed by levels taken as described in § 22. If detailed topography

is required it is usually taken by the cross-section method, the instruments used being the hand level, rod and tape. Five-foot contours are usually required and are determined by selecting such a position that a rod reading on center is obtained which puts the eye of the observer on a five-foot contour. Thus if the elevation of the station is 873.6 and the observer reads 1.4 on the rod, his *HI* is then 875.

The rod is then moved perpendicular to line until the reading is 5 feet when it is on the 870 contour and its distance from center is read and plotted. The rod is then moved to read 10 feet, and so on. In this way five-foot contours may be rapidly and accurately taken and plotted.

The points where the contours cross center line should also be determined for accuracy in plotting and for checking the elevations. The distances to contours should always be noted as a check on the plotting and the notes should, of course, run up the page as in Fig. 5.

All buildings and other structures near the line should be carefully located and bearings of fences, roads and streams determined and noted.

The preliminary survey is sometimes made by the stadia method, when the topography is picked up by side shots. Usually the stadia will be sufficiently accurate for the purpose if distances and vertical angles are read on both backsight and frontsight, though when the line is being run on ruling gradient it is somewhat safer to run levels over it. The method is especially valuable in rugged topography since the salient features can be picked up and more information gained in a given time than by the method given above if the country is sufficiently open to permit the stadia shots to be taken.

The organization of the party and detailed method of conducting the survey will vary with its character and purpose and the openness of the country. These subjects are fully discussed in various books on railroad location and will not be entered into here except to state that the Locating Engineer will take notes of all matters affecting the final location or cost of the line. He selects the ground ahead of the party by the same methods as were used on reconnoissance, supplemented by the use of the vertical circle in running uniform gradients. The topographer should also take complete notes

on the matters mentioned above in order to check the work of the engineer and insure as far as possible the collection of complete data.

40. The Location. — As soon as the map of the preliminary survey or a sufficient portion of it is complete, the Locating Engineer starts the projection of the final location. If on ruling or uniform gradient, a " grade contour " is very useful. This is plotted exactly the same as a general route on uniform gradient is put on the Geological Survey sheets (see §34), except that it must not be carried too far ahead of the paper location, as the corrected distances and possibly compensation for curvature will necessitate a revision of the grade elevations. This grade contour is, of course, a surface line, *i.e.*, a line of no cut or fill along the center line, and it will assist greatly in properly placing the trial location.

Profiles of trial projections are easily made up by noting where they cross the contours, and from these, or directly from the maps, quantities may be determined and costs estimated. Costs of structures on alternate projects need not be determined for comparisons unless they are so different as to require different structures.

The detailed comparison of the trial locations must include many factors aside from cost of construction. These factors involve a knowledge of the entire field of railroad engineering and hence cannot be discussed here.

As fast as the projected location is finished, the center line is staked on the ground as before described. Usually the transition curves, if used, are not run in, but offsets are allowed for them.

In staking the center line, property lines, roads, streams, etc., are located and right of way maps made up and filed as may be required. Deeds for right of way are obtained and the line is made ready for construction as described in Chapter II.

PROBLEMS.

1. A T. & S. aneroid barometer has a movable altitude scale which gives the following readings for different settings of the scale:

Inches.	Feet.	Feet.	Feet.	Feet.
31	0	190	380	570
30.5	445	637	825	1016
30	891	1081	1276	1471
29.5	1341	1537	1733	1929
29.	1800	1995	2199	2406
28.5	2270	2476	2680	2888
28	2751	2958	.....	.....

Show which of these settings would give results corresponding most closely with (30).

2. On the day the mercurial barometer readings of § 38 were taken, an aneroid was read at Sta. A some 400 feet lower and several miles distant as follows: At 8 A.M., 904 ft.; 1 P.M., 960 ft.

Find the height of Sta. B below Sta. A for a reading at Sta. B of 460 ft. at 9.45 A.M. with an air temperature of 71° F.

3. Obtain a Geological Survey sheet on which are two towns on the same side of a valley at least eight miles apart and plot a general route for a railroad on a uniform gradient.

4. An engineer having only one aneroid barometer took the following readings for rates of change.

8 A.M. 29.301	12 M. 29.407	5 P.M. 29.612
8.15 A.M. 29.309	12.30 P.M. 29.403	5.20 P.M. 29.613

With times for abscissas draw a curve of probable corrections.

CHAPTER IV.

Turnouts.

41. General. — A turnout is a track for passing from the main track to a siding or other track. It consists of three parts as shown in Fig. 15; the movable switch or switch rails AD and BE for turning from the main track, the lead rails

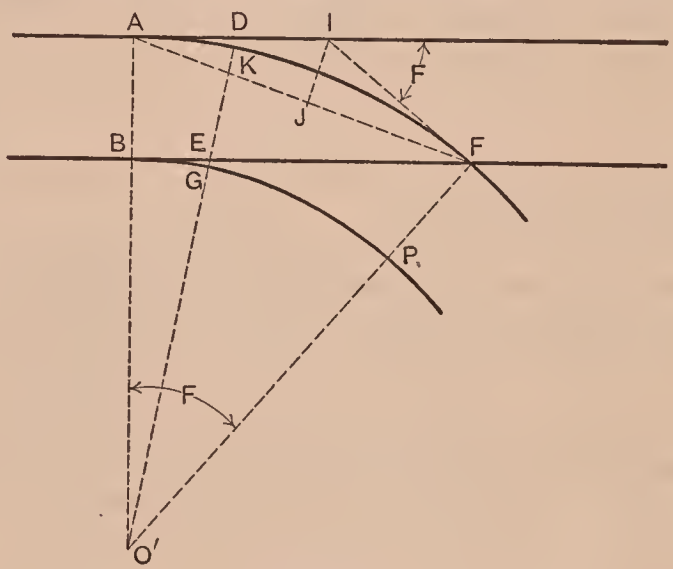


Fig. 15.

KF and GP for leading to the frog, and the frog at F to provide open flangeways for wheels to pass the intersection on either track.

The movable ends of the switch rails, which may be at A and B or at D and E, are controlled from a switch stand placed outside the track on a long tie called the head-block. The distance from the head-block to the point of frog, measured along the main track, is called the lead; stub lead if the head block is at D as for the stub switch, and point lead if at A as for the point switch.

The point of switch is at A, which is the heel of the stub switch rail and the toe or point of the point switch rail.

The throw is the distance the toe of the switch rail is moved in opening or closing the switch. The spread is the distance

between main and switch rail centers at the heel of the point switch. Some 5 to $5\frac{3}{4}$ inches is required to give clearance for wheel flanges.

The gage of a track is the width between the rail heads measured at points $\frac{5}{8}$ of an inch below the tops of the rails.

The ordinary turnout frogs are numbered, the number being the axial length from the theoretical point, or intersection of the sides produced, to where the width is unity. Since this length is one-half the cotangent of the frog angle F, the frog number,

$$n = \tfrac{1}{2} \cot \tfrac{1}{2} F. \tag{31}$$

42. Stub Switch Turnout from Straight Track.— In the stub switch, Fig. 15, the ends of the switch rails back of A and B are spiked down so that in throwing the free ends to K and G for the turnout they are bent to a curve. On this account it is customary to assume a circular arc for the turnout curve, although the frog is usually straight except for street railroad track.

Given the frog angle F, the throw t, and the gage g; to find the radius of the turnout, r, the point lead and the length of the switch rail l.

In the triangles AFO' and ABF,

$$r + \frac{1}{2} g = \frac{g}{\text{vers } F}. \tag{32}$$

$$\left.\begin{aligned} \text{Point lead, } BF &= (r + \tfrac{1}{2} g) \sin F, \\ \text{or, } &= g \cot \tfrac{1}{2} F. \end{aligned}\right\} \tag{33}$$

Similarly in the triangle ADO' (neglecting the effect of gage),

$$\text{Vers } AO'D = \frac{t}{r},$$

$$l = r \tan AO'D. \tag{34}$$

Since for small angles tangent offsets increase approximately as the squares of the tangents,

$$l = BF \sqrt{\frac{t}{g}}. \tag{35}$$

These values for laying out the turnout are usually given in terms of the frog number n.

Thus from (31) and (33),

$$BF = 2\,gn. \tag{36}$$

From the triangle BFO',

$$(FO')^2 = (BF)^2 + (BO')^2, \quad \text{or}$$
$$(r + \tfrac{1}{2}\,g)^2 = (2\,gn)^2 + (r - \tfrac{1}{2}\,g)^2.$$

Solving,

$$r = 2\,gn^2. \tag{37}$$

From (35), (36) and (37),

$$l = 2\,n\,\sqrt{gt} = \sqrt{2\,rt}. \tag{38}$$

For the standard gage of 4 feet $8\frac{1}{2}$ inches and a throw of 5 inches, these formulas become

$$\left.\begin{array}{ll} \text{Lead, } BF & = 9.42\,n, \\ \text{Radius, } r & = 9.42\,n^2, \\ \text{Switch rail, } l & = 2.8\,n. \end{array}\right\} \tag{39}$$

The lead rail may be laid out by ordinates from the main rail AI proportional to the squares of the distances from A, giving at the quarter, half and three-quarter points, $\frac{1}{16}\,g$, $\frac{1}{4}\,g$, and $\frac{9}{16}\,g$, respectively. Or, the middle ordinate of the chord $AF = (r + \frac{1}{2}\,g)$ vers $\frac{1}{2}F$, and the side or quarter ordinates will be three-quarters the middle ordinate.

The circular arc assumed in developing these formulas was approximately secured when cast-iron frogs were in use. At present much longer frogs are made up from rail sections, and usually without curvature in steam railroad practice. Again, 22 feet is about the maximum allowable distance for an unspiked switch rail, and this corresponds to about a No. 8 frog; so that for large frog numbers the switch rail will not coincide with the curve, while for small ones the frogs will not coincide unless the leg is bent.

On the other hand, variations of 10 per cent are frequently made in the lead to avoid cutting a rail or for other reasons, with but slight effect upon the riding qualities of the turnout

43. Stub Switch Turnout from Curved Track.— The lead and length of switch rail are practically unaffected for

curved main track until the curvature becomes quite sharp. The degree of curve is increased or decreased by that of main track according as the turnout is on the inside or on the outside of the curve, and the lead rail ordinates can be used as in §42. On this account and because the stub switch has practically gone out of use, at least for main line, the derivation of the accurate formulas is omitted.

44. Stub Switch Turnout with Tangent at Frog. — The effect of the tangent l' at the frog F, Fig. 16, so far as the

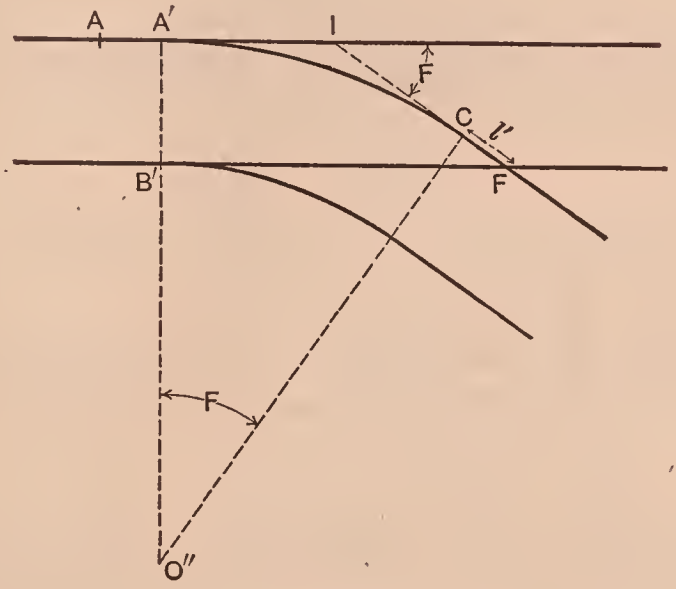


Fig. 16.

turnout up to the tangent point C is concerned will be to replace the gage by $(g - l' \sin F)$.

Then from (36), (37) and (38),

$$\left.\begin{aligned} \text{Lead, } BF &= 2n(g - l' \sin F) + l' \cos F, \\ \text{Radius, } r &= 2n^2 (g - l' \sin F), \\ \text{Switch rail, } l &= 2n\sqrt{t(g - l' \sin F)} = \sqrt{2rt}. \end{aligned}\right\} \quad (40)$$

The effect is to shorten the lead by the length of the tangent, *i.e.*, $AA' = l'$, and to sharpen the curve. The frog is usually laid out straight, as already indicated.

45. Point Switch Turnout from Straight Track.— In the point switch, the outside rail of the main track and the inside rail of the turnout are continuous past the switch rails, a slight kink in each protecting the sharp points of the straight (on the gage side) switch rails. This gives a continuous rail support for the wheels for either main line or turnout, and

prevents the jolting at the toe, so characteristic of the stub switch. It is also less sensitive to relative movement of rails due to creeping or temperature changes. For these reasons it has practically replaced the stub switch, although rather more sensitive to disturbance by ice and snow.

In Fig. 17, AK is the straight switch rail in position for the turnout curve KF and making the angle S, called the

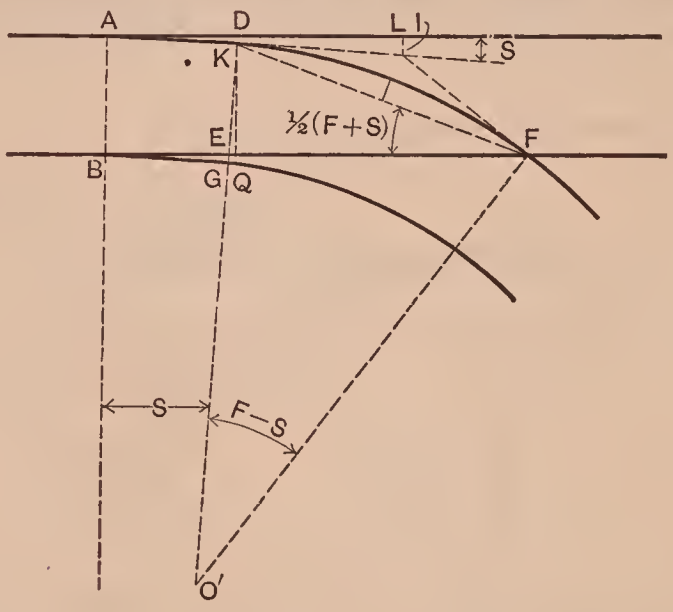


Fig. 17.

switch angle, with the main track. The long chord KF makes an angle $= \frac{1}{2}(F - S)$ with the switch rail AK, or $\frac{1}{2}(F + S)$ with the main rail.

In the triangle DQF,

$$KF = \frac{g - t}{\sin \frac{1}{2}(F + S)}.$$

But

$$r + \frac{1}{2}g = \frac{KF}{2 \sin \frac{1}{2}(F - S)}, \quad \text{or}$$

$$\left.\begin{aligned} r + \frac{1}{2}g &= \frac{g - t}{2 \sin \frac{1}{2}(F + S) \sin \frac{1}{2}(F - S)} \\ &= \frac{g - t}{\cos S - \cos F}. \end{aligned}\right\} \quad (41)$$

The last value is found by substituting for $2 \sin \frac{1}{2}(F + S) \sin \frac{1}{2}(F - S)$ its equal $(\cos S - \cos F)$, and will be conven-

ient for field computations with natural functions, while the first form is better for use with logarithms.

$$\sin S = \frac{t}{l}.$$

$$BF = l \cos S + KF \cos \tfrac{1}{2}(F + S), \text{ or}$$

$$\text{Point lead, } BF = l \cos S + (g - t) \cot \tfrac{1}{2}(F + S). \quad (42)$$

With a tangent l' at F the gage would be replaced as before, giving

$$\left.\begin{aligned} r + \tfrac{1}{2}g &= \frac{g - t - l' \sin F}{2 \sin \tfrac{1}{2}(F + S) \sin \tfrac{1}{2}(F - S)} \\ &= \frac{g - t - l' \sin F}{\cos S - \cos F}, \\ BF &= l \cos S + (g - t - l' \sin F) \\ &\quad \cot \tfrac{1}{2}(F + S) + l' \cos F. \end{aligned}\right\} \quad (43)$$

The middle ordinate for the chord from the heel K to the frog, point or toe as the case may be, will be

$$m = (r + \tfrac{1}{2}g) \operatorname{vers} \tfrac{1}{2}(F - S). \quad (44)$$

46. Point Switch Turnout on the Inside of Curved Track. — With curved main track the gage lines of the switch rails have the curvature of the track, so that the turnout at the heel of the outer switch rail makes the switch angle S with the main track the same as for straight track.

The center of the turnout will thus be at O', Fig. 18, on the line KO', making the switch angle S with the radius KO. For convenience use the radius r of the outer rail of the turnout and R' of the inner rail of the main track.

In the triangle KOO', $KO' = r$; $KO = R' + a$, where $a = g - t$; angle $OKO' = S$; $OO' = b$.

By trigonometry,

$$b^2 = r^2 + (R' + a)^2 - 2r(R' + a) \cos S.$$

In the triangle FOO', $FO' = r$; $FO = R'$; angle $OFO' =$ frog angle F,

$$b^2 = r^2 + R'^2 - 2rR' \cos F.$$

Placing the two values of b^2 equal,

$$R'^2 + 2aR' + a^2 - 2r(R' + a) \cos S = R'^2 - 2rR' \cos F.$$

Radius of outer rail, $r = \dfrac{aR' + \frac{1}{2}a^2}{(R' + a)\cos S - R'\cos F}$. (45)

For the angles at O and O' in the two triangles use $\tan \frac{1}{2}(A - B) = \dfrac{a - b}{a + b} \tan \frac{1}{2}(A + B)$.

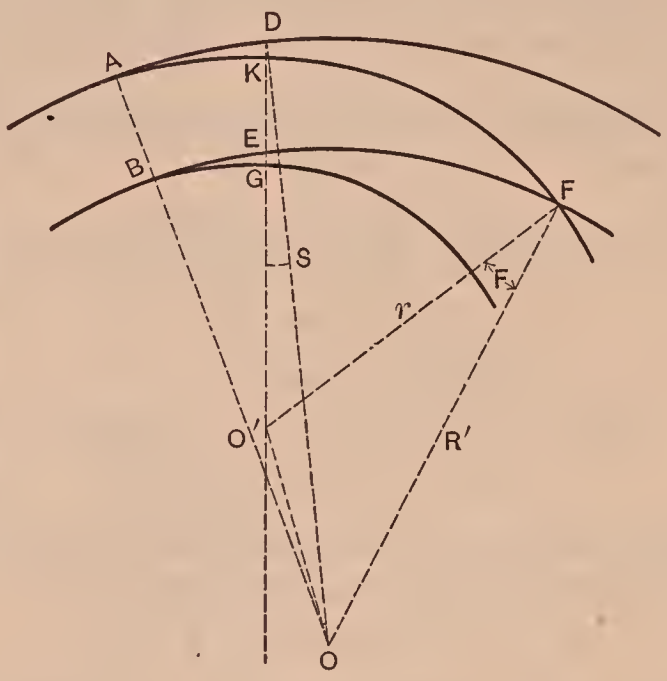


Fig. 18.

The difference of those at O will give the central angle for the stub lead, and of those at O' that for the length of the turnout from heel to frog.

Example. — Find the data for laying out a turnout with a no. 10 frog, $F = 5° 43\frac{1}{2}'$, on the inside of a 4° curve; 18-ft. switch rail, $5\frac{1}{2}$ in. spread, giving $S = 1° 27\frac{1}{2}'$.

$R_4 = 1432.5$; $R' = 1430.15$; $a = 4.71 - 0.46 = 4.25$.

Solution.

$$(45),\quad r = \frac{4.25 \times 1430.15 + \frac{1}{2}(4.25)^2}{(1430.15 + 4.25)\,0.99968 - 1430.15 \times 0.99501}$$

$$= \frac{6087.1}{10.94} = 556.40.$$

Triangle FOO'.

$FO - FO'$	$= 1430.15 - 556.40 = 873.75$ ...	2.94138
$FO + FO'$	$= 1430.15 + 556.40 = 1986.55$ (AC)	6.70190
	$\frac{1}{2}F = 2° 52'$, cot.....	1.30038
$\frac{1}{2}(O_1' - O_1)$	$= 83° 30'$, tan.....	0.94366

$O_1' = 170° 38'$; $O_1 = 3° 38'$.

Triangle KOO'.

$KO - KO' = 1434.40 - 556.40 = 878.00 \ldots\ldots 2.94349$

$KO + KO' = 1434.40 + 556.40 = 1990.80\ (AC)\ 6.70097$

$\frac{1}{2} S = 0° 44'$, cot $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.89280$

$\frac{1}{2}(O_2' + O_2) = 88° 20'$, tan $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.53726$

$O_2' = 177° 36'; O_2 = 0° 56'.$

$KOF = O_1 - O_2 = 2° 42'$; stub lead $EF = 162'/240\frac{1}{2}' = 67.4$ ft.

$KO'F = O_2' - O_1' = 6° 58'; KF = 418'/618' = 67.6$ ft.

47. Point Switch Turnout on the Outside of Curved Track. — For convenience use the radius r of the outer rail of the turnout and R'' of the outer rail of the main track, with $a = g - t$.

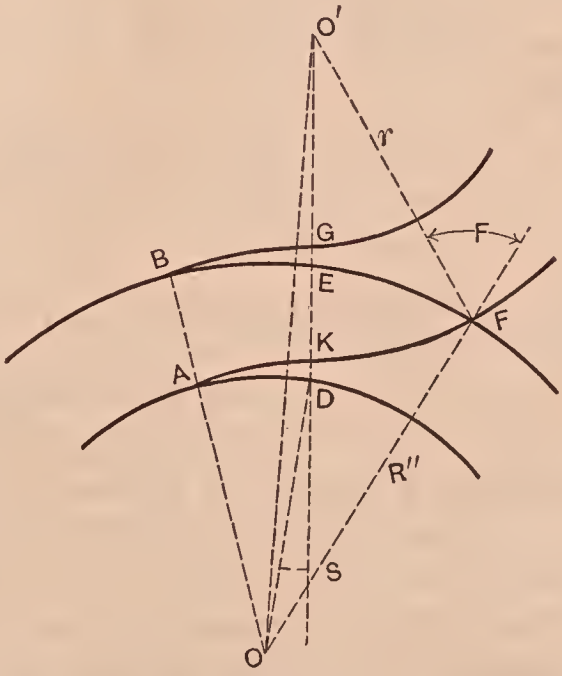


Fig. 19.

In the triangle KOO', Fig. 19, as in § 46,

$$b^2 = r^2 + (R'' - a)^2 + 2\,r\,(R'' - a)\cos S.$$

In the triangle FOO'

$$b^2 = r^2 + R''^2 + 2\,rR''\cos F.$$

Equating,

$$-2\,aR'' + a^2 + 2\,r\,(R'' - a)\cos S = 2\,rR''\cos F.$$

$$r = \frac{aR'' - \frac{1}{2}a^2}{(R'' - a)\cos S - R''\cos F}. \qquad (46)$$

Having r, the angles at O and O' are found as in the preceding section.

EXAMPLE. — Find the data for laying out a turnout with a no. 6 frog, $F = 9° 31\frac{1}{2}'$, on the outside of an 8° curve; 15-ft. switch rail, $5\frac{1}{2}$ inch spread, giving $S = 1° 45'$; $R_s = 716.25$; $R'' = 718.60$; $a = 4.71 - 0.46 = 4.25$.

Solution.

$$(46), \quad r = \frac{4.25 \times 718.60 - \frac{1}{2}(4.25)^2}{(718.60 - 4.25)\,0.99953 - 718.60 \times 0.98622}$$

$$= \frac{3045.05}{5.32} = 572.37.$$

Triangle FOO'.

$FO - FO'$	$= 718.60 - 572.37 = 146.23$.....	2.16504
$FO + FO'$	$= 718.60 + 572.37 = 1290.97\ (AC)$	6.88908
$\frac{1}{2}(O_1' + O_1)$	$= 4° 46'$, tan.....	8.92110
$\frac{1}{2}(O_1' - O_1)$	$= 0° 32\frac{1}{2}'$, tan.....	7.97522
	$O_1' = 5° 18\frac{1}{2}'$; $O_1 = 4° 13\frac{1}{2}'$.	

Triangle KOO'.

$KO - KO'$	$= 714.35 - 572.37 = 141.98$.....	2.15223
$KO + KO'$	$= 714.35 + 572.37 = 1286.72\ (AC)$	6.89052
$\frac{1}{2}(O_2' + O_2)$	$= 0° 52\frac{1}{2}'$, tan.....	8.18390
$\frac{1}{2}(O_2' - O_2) = 0° 6'$, tan		7.22665
$O_2' = 0° 58\frac{1}{2}'$; $O = 0° 46\frac{1}{2}'$.		

$KOF = O_1 - O_2 = 3° 27'$; stub lead $EF = 207/478\frac{1}{2} = 43.2$ ft.
$KO'F = O_1' - O_2' = 4° 20'$; lead rail $KF = 260/601 = 43.3$ ft.

48. Point Switch Turnouts by Tables.—Table XI, from Camp's Notes on Track, gives the data for laying out turnouts from straight main track without computation.

For curved main track he finds that the lead should be increased, or decreased, by $Dn^2/144$, according as the turnout is from the inside or from the outside of the curve. D is the degree of the main track curve and n is the frog number.

For the middle and quarter point ordinates from the outer rail of the turnout curve to the chord joining the heel with the point of frog the ordinates for straight main track are divided by the degree of turnout curve for straight track and multiplied by the degree of curve for main track. One and a quarter times these values are added if the turnout is on the inside, or one and an eighth times are subtracted if the turnout is on the outside.

Within the ordinary range of frog numbers and degrees of

curve it is claimed that these corrections will give results checking the true values quite closely.

49. Three Throw Point Switch Turnout.— If the main line frogs are opposite and the turnouts symmetrical, as

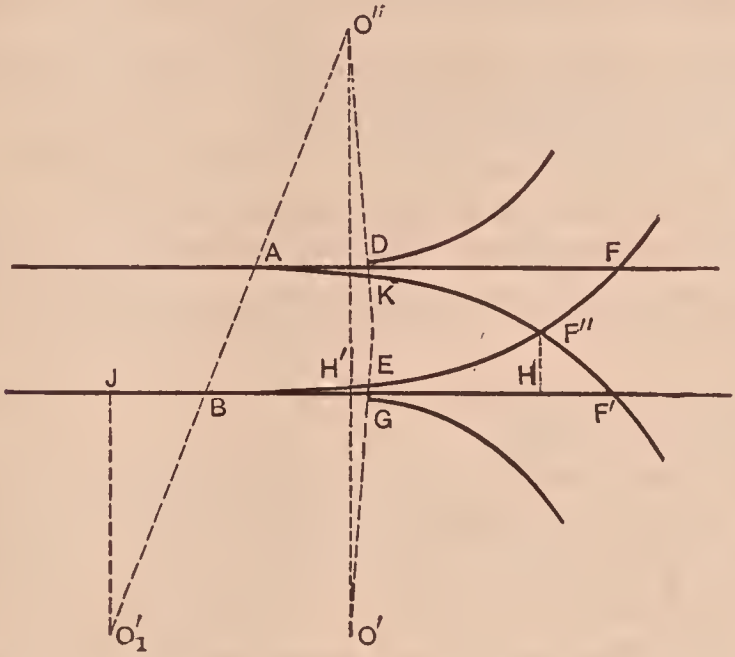


Fig. 20.

assumed in Table XI and shown in Fig. 20, $\frac{1}{2}\,g$ and $\frac{1}{2}\,F''$ would replace g and F in the second members of (41) and (42), giving

Point lead to crotch frog,

$$\left.\begin{aligned} BH &= l\cos S + (\tfrac{1}{2}g - t)\cot\tfrac{1}{2}\,(\tfrac{1}{2}F'' + S), \\ \cos\tfrac{1}{2}\,F'' &= \cos S - \frac{\tfrac{1}{2}g - t}{r + \tfrac{1}{2}g}\,. \end{aligned}\right\} \quad (47)$$

If the main line frogs are not opposite, or if the turnouts are not symmetrical, the data given would locate the centers O' and O'', and the problem would be a crossing frog problem which could be worked in a manner similar to those of §45 and §46.

Thus if O' were moved to O_1', the distances $O_1'\,J$, JH' and $H'\,O''$ would be known, locating the centers.

50. Turnouts without Instruments, Formulas or Tables. — In making out a set of turnout standards and in many other cases it will be desirable to make use of the formulas and table given for laying out turnouts. Sometimes, however, the following graphic methods for making a direct solution on the track will be found helpful.

Thus for the stub switch of Fig. 15, lay off $4n + \frac{1}{2n}$ (Eq. 1.) on the gage line of the rail to the left from F, then locate a point 4 feet from this and $4n + \frac{1}{2n}$ from F and join with F for the frog tangent. Produce this line by tape or by eye to I for the PI of the turnout curve.

Lay off IF on the main rail to locate the heel at A. Draw the chord AF and drop the perpendicular IJ. This perpendicular is bisected by the turnout curve, and the side ordinates from the chord will be three-fourths of the middle ordinate.

A tangent l' at the frog, as in Fig. 16, will change the outside lead rail chord to $A'C$.

For the point switch, Fig. 17, after laying out the frog tangent as before, estimate the distance from I to the main rail and measure FI; multiply $(FI + l)$ by the spread of the point rail per foot (t/l) and compare with LI. A second trial should suffice, as the frog angle is greater than the switch angle and an exact equality of the tangents is not essential.

If the track is curved at the frog, measure $4n + \frac{1}{2n}$ each way from the frog point, draw the chord and measure the middle ordinate; add or subtract this ordinate or tangent offset as the case may be, in laying off the frog tangent IF. The same method can be used in laying off the tangent to a curved switch rail.

If for any reason the lead for a circular curve is changed, making the tangents unequal, a parabola connecting these unequal tangents can be laid out by joining the vertex with the center of the chord and drawing the side ordinates parallel with this center one; the lengths being obtained as in Fig. 15 for the circle. Or, the length of the shorter tangent can be laid off on the longer, the circular arc drawn, and the excess left as a tangent to the circle.

For complicated turnout and crossing problems these field methods can be adapted to the draughting room, using a scale of from 5 to 10 feet per inch and checking the relation between distance, central angle and curvature by computation for increased accuracy.

PROBLEMS.

1. Find the lead and radius for a 5-ft. tangent at the frog of a point switch turnout for a no. 8 frog.

2. Find the lead and radius for a 5-ft. tangent at the frog of a point switch turnout on the outside of a 4° curve for a no. 7 frog.

3. Find the data for staking out a crossover between two straight tracks, 12 ft. centers, for stub switches, no. 10 frogs, the turnout curves being continued beyond the frogs to a reversing point.

4. Two turnouts on double track, 13 ft. centers, are connected by a tangent between frogs so as to form a crossover from one track to the other.

Find the data for laying out with no. 11 frogs and 20-ft. point switch rails with 6-in. spreads.

5. On location a 4° curve intersects the first rail of a single track tangent at a deflection angle of 28° 32′.

Find the frog angles and the distances between frog points on both tracks for a single track grade crossing.

6. Compare the radius and lead of a stub switch turnout for a no. 9 frog with those of a point switch having a 15-ft. switch rail and a $5\frac{1}{2}$-in. spread.

7. Check the rule of §48 for lead and middle ordinate for a point switch turnout on the inside of a 3° curve by comparison with the values computed by §46; no. 11 frog; 18-ft. switch rail, and $5\frac{1}{2}$-in. throw.

8. Find the crotch frog angle and lead for a 10-ft. tandem three throw point switch turnout with no. 9 frogs and 20-ft. switch rails.

9. Find the middle and side ordinates from the turnout chord AF for a stub switch turnout on the inside of a 3° curve, using a no. 10 frog.

10. For a stub switch turnout with a no. 12 frog, compare the theoretical lead and radius with the actual, if the lead rail curve is tangent to the switch rail at the toe and the free portion of the switch rail is an arc of a circle 22 ft. long.

CHAPTER V.

Special Problems.

51. Middle Ordinates for curving Rails. — In Fig. 21, let C be the center; AD, a diameter and FE, or c, a chord of the simple curve FAE whose middle ordinate is AB, or m.

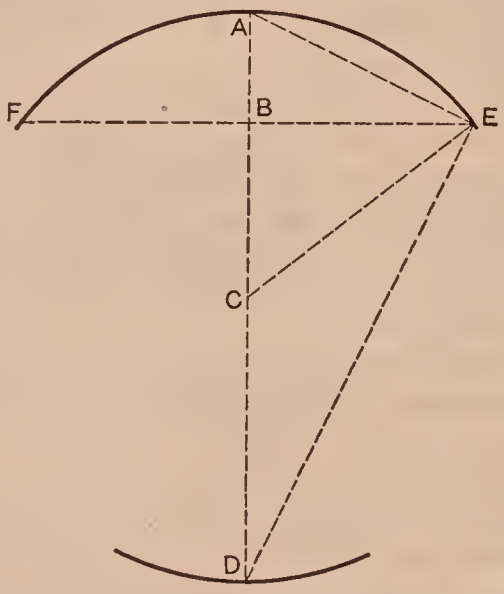


Fig. 21.

In the right triangles ABE and ADE, the angles AEB and ADE are equal, being measured by one-half the equal arcs AF and AE. Therefore the triangles are similar and

$$AB : AE = AE : AD.$$

But for flat arcs AE is practically $\frac{1}{2}\ FE$, or $\frac{1}{2}\ c$. Substituting and reducing,

$$m = \frac{c^2}{8\ R} = \frac{c^2 D}{45\,840}\ \text{(approx.)}. \qquad (48)$$

The use of this formula may be illustrated by determining the length of the chord whose middle ordinate in inches equals the degree of the curve. Since m in the formula is in feet, $\frac{D}{12}$ will be its value in the assumed problem.

Substituting this value for m

$$c = 61.8 \text{ feet.}$$

That is, to find the degree of a curve approximately, determine the length in inches of the middle ordinate for a chord of 61.8 feet. This fact is useful in determining the degree of curve on old track, and the formula is also used to find the middle ordinate for curving rails to a given radius. Table XII was computed by its use.

It is also used to determine the length of chord whose middle ordinate equals the desired elevation of outer rail on curves, say $\frac{3}{4}$ inch per degree of curve, etc.

52. Correction for Curvature and Refraction in Leveling.—In applying § 51 to this problem, the radius is the mean radius of the earth, r (= 20 890 129 feet), and the length of sight, L, is one-half the chord c, giving

$$\text{Correction} = \frac{L^2}{2r} = \frac{L^2}{41\,780\,258}. \tag{49}$$

The effect of refraction is to reduce this, since it makes the line of sight slightly concave downward. The radius of the line of sight varies, due to variations in refraction, but averages about $7r$, giving for the correction

$$C' = \frac{3L^2}{7r} = \frac{L^2}{48\,743\,634}. \tag{50}$$

This correction is subtractive from a rod reading or additive to elevation. It amounts to 0.020 feet in 1000 and varies as the square of the distance.

The correction should be applied when a long frontsight is necessary, as in crossing a stream, but the results should not be depended upon for accurate work without checking by a backsight on account of the uncertainty in refraction.

53. Elevation of Outer Rail on Curves. — The outer rail on curves is made higher than the inner one, so that the weight will have a component towards the center of the curve which will tend to balance the centrifugal force. From Mechanics, —

$$\text{Centrifugal force} = \frac{Mv^2}{R}, \tag{51}$$

where M is the mass; v, velocity and R, radius of curvature, all in the foot-pound-second system.

And on an inclined plane making an angle α with the horizontal,

Component of weight along plane $= G \sin \alpha$, (52)

where G is weight in pounds.

Let e be the elevation of outer rail and g' the distance between centers of rails, both in feet.

Then $$\frac{e}{g'} = \sin \alpha,$$

also $$M = \frac{G}{32.16},$$

$$v^2 = \frac{V^2\,(5280)^2}{(3600)^2},$$

where V is in miles per hour.

Equating the expressions for the two forces for equilibrium, substituting the above values and solving,

$$e = \frac{0.066\,86\ V^2 g'}{R} = 0.000\,011\,7\ V^2 g' D. \qquad (53)$$

The distance g' is slightly greater than the gage of the track, but since the rail can be elevated for only one speed, which may never be the exact speed of the train, nothing is gained by splitting hairs and the gage will be used for g'.

For the standard gage of 4 feet $8\frac{1}{2}$ inches (53) becomes

$$e = \frac{0.315\ V^2}{R} = 0.000\,055\ V^2 D. \qquad (54)$$

For the elevation in inches ($E = 12\ e$),

$$E = \frac{3.78\ V^2}{R} = 0.000\,66\ V^2 D. \qquad (55)$$

Table XIII, from the Manual of Recommended Practice of the Am. Ry. Eng. and M. of W. Assoc., was computed by this formula.

When tracks are used for only one kind of service, so that the speed on a given curve is nearly constant, it is possible to apply the proper elevation for that speed. In the usual

case, with varying speeds, it is customary to elevate for nearly the highest one, or for fast passenger trains, on account of safety. With the centrifugal force balanced there is still a flange pressure on the front outer wheels of the trucks due to the use of parallel axles.

Equation (55) may be used with (48) for finding the chord whose middle ordinate in inches will equal the proper elevation of outer rail, for the convenience of the trackmen.

The Manual mentioned above also gives the following recommendations: —

" Ordinarily an elevation of 8 inches should not be exceeded and the speed of trains should be regulated to conform to that elevation.

" The elevation of curves should be zero at the point of spiral (or transition curve) and should increase to full elevation at the end of the spiral or beginning of the simple curve.

" In ordinary practice it is recommended that the elevation be run out at the rate of 1 inch in 60 feet, but this will be modified by the same conditions that should vary the length of the easement curve used.

" The inner rail should be maintained at grade."

54. Vertical Curves. — Vertical curves are necessary at changes of gradient in order to avoid undue shock or slackening of the couplings.

Theoretically, their length should depend upon the total change of gradient and the length of the longest train. Practically, the length is usually found by dividing the total change of gradient by the allowable change per station, though some roads use an arbitrary length, say 400 feet, for all cases.

The Am. Ry. Eng. and M. of W. Assoc. recommends a rate of change of 0.1 per cent per station on summits and 0.05 per cent per station in sags for first-class lines and double these values for minor lines.

The gradient of the curve at any point is that of its tangent at that point. For a station length, the average gradient is that of the tangent at its middle point, or its chord which is parallel to that tangent. Hence the change of gradient per station is the chord offset of the curve. That is, suppose the chords for two consecutive stations have gradients of + 0.4 per cent and + 0.6 per cent, respectively. The rate of change of gradient is 0.2, and if the chord for the first station length be

extended over the second the chord offset will be 0.2. The tangent offset per station will be 0.1, since, as shown in Chapter I, the tangent offset is one-half the chord offset.

The degree, or radius, of a vertical curve is never found, as they are easily laid out by the tangent offset method, using the fact developed in the last paragraph, *i.e.*, that the tangent offset per station is one-half the rate of change of gradient.

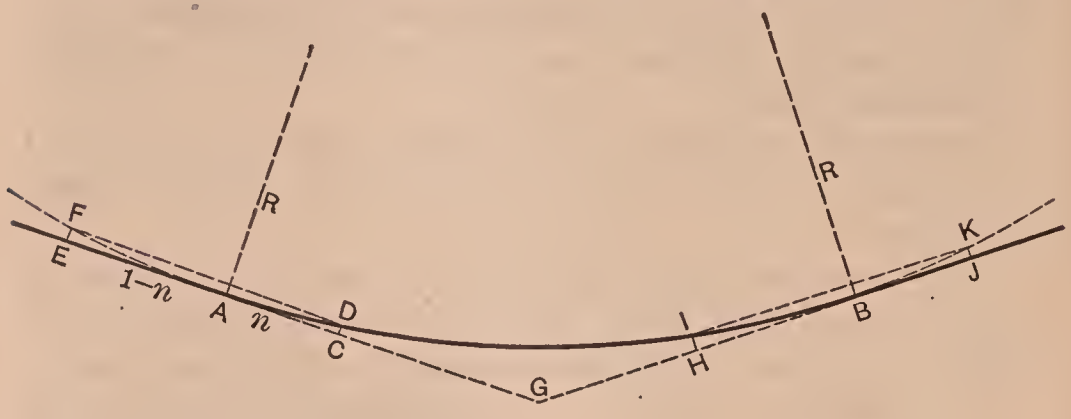


Fig. 22.

Fig. 22 shows the general case of a vertical curve beginning and ending between stations. The two grade lines, EG and GJ, meeting at G are connected by the vertical curve $ADIB$. The beginning of the curve, A, lies between the regular stations C and E. Call its distance from C, n, in stations. AE will then be $1 - n$. Extend the curve backward to F.

Let t be the tangent offset per station ($= \frac{1}{2}$ the change of gradient), then

$$DC = n^2\, t \text{ and}$$

$$FE = (1 - n)^2\, t.$$

The difference between these two offsets applied to the gradient of EG will give the gradient of FD which is one chord of the curve. The gradient of the succeeding chords can then be found in turn by applying the rate of change of gradient per station.

To find the grade elevation of D, first find that of C and add DC. This gives a starting point for the grade elevations of the regular stations, and having the gradients of the chords the others are readily computed. A similar process serves to get from curve to tangent at the other end, and the grade of B should always be figured both along the curve and along the tangents, thus checking the computations.

EXAMPLE. — A -0.6 per cent gradient and a $+0.4$ per cent intersect at sta. $186+30$ with a grade elevation of 217.35. Determine grade elevations at the regular stations on a vertical curve changing gradient at the rate of 0.2 per cent per station.

Solution. — Total change of gradient $= +0.4 - (-0.6) = 1.0$ per cent.

$$\text{Length of curve} = \frac{1.0}{.2} = 5 \text{ stations.}$$

$$PC = 186+30-(2+50) = 183+80.$$
$$PT = 186+30+(2+50) = 188+80.$$
$$n = .20; \quad 1-n = .80.$$

Grade of $PC = 217.35 + 2\frac{1}{2} \times 0.6 = 218.85.$

Grade of $PT = 217.35 + 2\frac{1}{2} \times 0.4 = 218.35.$

Offset at 184, $0.2^2 \times 0.1 = 0.004$

Offset at 183, $0.8^2 \times 0.1 = 0.064$

Difference $= 0.06$

Gradient of chord FD $= -0.66$ per cent.

Grade of 184 on tangent, $218.85 - .2 \times 0.6 = 218.73.$

Grade of 184 on curve, $218.73 + .004 = 218.734.$

Sta.	Gradient.	Grade.
183+80, *PC*	.....	218.85
184	−0.46	218.734
185	−0.26	218.274
186	−0.06	218.014
187	+0.14	217.954
188	.....	218.094
+80, *PT*		218.35

☞ $188+80$ *PT*, $218.094 - .064 + 0.8 \times 0.4 = 218.35$, checking the above value.

Vertical curves can usually be made to begin and end at regular stations, when the above method will be more easily applied, since the offsets need not be computed. That is, simply change the gradient of the first chord by one-half the rate of change per station and the others by the full rate. This should give a grade checking the *PT*, and the gradient of the last chord should differ from that of the tangent by one-half the rate of change per station.

For the general case most engineers prefer another method in which the tangent offsets from the first tangent for the entire curve are figured by making them proportional to the squares of the distances from the *PC*. This method will be applied to the example given above.

Sta.	Grade on Tang.	$t' = n^2 t$.	Grade on Curve.
183 + 80, *PC*	218.85	.....	218.85
184	218.73	.004	218.734
185	218.13	.144	218.274
186	217.53	.484	218.014
187	216.93	1.024	217.954
188	216.33	1.764	218.094
+ 80, *PT*	215.85	2.50	218.35

55. The Stadia Method. — This will be found so useful in railroad work that it should be reviewed and the following formulas derived:

$$D = kr \cos^2 \alpha + (f + c) \cos \alpha, \tag{56}$$

$$H = kr \cos \alpha \sin \alpha + (f + c) \sin \alpha, \tag{57}$$

where D is the horizontal distance; H, the difference of elevation; k, the stadia constant; r, the rod reading or intercept; α, the angle of inclination; f, the focal length of the objective and c, the distance from the center of the objective to the center of the instrument.

For horizontal sights (56) becomes

$$D = kr + f + c. \tag{58}$$

Subtracting (56) from (58),

$$\left.\begin{aligned} \text{Correction} &= kr\,(1 - \cos^2 \alpha) + (f + c)\,(1 - \cos \alpha) \\ &= kr \sin^2 \alpha \text{ (nearly).} \end{aligned}\right\} \tag{59}$$

This correction, subtracted from the distance found by neglecting the slope, will give the true distance as accurately with three places in the computation as the direct method will with five up to 6°, or with four up to 18°.

The vertical distance is the altitude of the triangle of base D, *i.e.*,

$$H = D \tan \alpha. \tag{60}$$

A diagram made up as described in the Jour. W. Soc. of Eng., Vol. III, p. 1399, will give both horizontal and vertical distance with one entry. For occasional field use, Table XIV, giving corrections to horizontal distances computed by (59), and natural tangents for differences of elevation, will be found more portable and convenient.

For accurate work the constant k should be determined by taking readings for various measured distances on level ground and solving for k in (58), f and c being measured on the instrument. In using Table XIV, multiply the correction from the table by the rod reading, or intercept, in feet, corrected for constant if necessary, and subtract the result from 100 times the (corrected) rod reading. For vertical angles over 18° subtract 0.1 ft. from the final result for $(f + c)\ (1 - \cos \alpha)$ in computing to tenths of feet.

56. Adjustment of Instruments.— The adjustments of the instruments should be taken up in the order given below. Before using an instrument, however, and especially before attempting to adjust it, the observer should focus the eyepiece upon the wires. Otherwise, errors due to parallax will enter into the work and make a good adjustment impossible.

This is done by focusing the eyepiece sharply on the wires while the field is made clear of any image by pointing at the sky or a piece of blank paper or even by throwing the objective out of focus. The focusing should be tested by sighting upon some object and noting if there be any relative motion of wires and image as the eye is moved and repeated if necessary.

In correcting the errors it is best to under-adjust rather than over-adjust, and care should always be taken that the screws have an even and firm bearing. In using two opposing screws, one should always be slightly loosened before the other is tightened. In adjusting the line of collimation care must be taken not to disturb the verticality of the vertical wire. In putting in the reticule, or in any case where the adjustments are badly in error, they must be gone through with roughly before attempting the final adjustment.

a. Transit Adjustments.

1. *Plate Levels.* — Level carefully, being sure that the levels are parallel with the leveling screws. Swing 180° in azimuth, thus reversing the levels and doubling their errors.

Correct one-half the apparent error by the use of the adjusting screws, bring bubbles to the center by the leveling screws and repeat.

It is best to work with one level at a time, beginning with the one having the larger error. Also rotate by use of the upper motion, as it is more important to have the levels correct for this motion if there is any difference between the two.

2. *Vertical Wire.* — If this is truly vertical it will cover a point upon which the transit has been set as the telescope is moved slightly up and down. Test in this way and correct if necessary by loosening the pairs of screws holding the reticule and turning it until the wire stands the test.

3. *Line of Collimation.* — Set up at about the middle of a level stretch at least 400 feet long. Backsight on a well-defined point and, plunging, set a point ahead. Swing in azimuth to the backsight. Plunge again, and correct one-fourth the apparent error, if any, by moving the vertical wire. Repeat the test and correction until the same point is obtained on frontsight whether the backsight is taken with the telescope direct or reversed.

4. *Horizontal Axis.* — Pick out a well-defined, high point on a building and set up so as to give as large a vertical angle as convenient for use. Drop down and line in a point near the ground. Swing 180° in azimuth, plunge and repeat. If the two lower points do not coincide, adjust the movable end of the telescope axis by trial until a point half-way between them is obtained in dropping down from the upper point.

5. *Telescope Level.* — This is made parallel to the line of collimation by the " two-peg " method. Set up half-way, by pacing, between two pegs or other solid points and read a rod on each with the bubble centered in each case. The difference of rod readings gives the true difference of elevation even if the level is not in adjustment, since the inclination of the line of sight would be the same in both cases and, with equal distances, would give equal errors which neutralize each other.

Now set up near one of the points and take a reading upon it. Apply the difference of elevation to this reading, thus obtaining a correct reading for the far point. If this reading is not obtained with the bubble centered, set the wire upon it and bring the bubble to the center by the adjusting screws.

If in error, the rod reading on the near point should be checked before making the final adjustment.

6. *Vernier of Vertical Circle.* — With the bubble of the telescope level centered, and the plates carefully leveled, set the vernier of the vertical circle at zero.

b. Y-Level Adjustments.

1. *Horizontal Wire.* — The horizontal wire is made truly horizontal by a similar method to that used with the transit for the vertical wire (see above).

2. *Line of Collimation.* — Sight on a well-defined point with the intersection of the wires after having loosened the clips so that the telescope is free to rotate in the Y's. Both leveling screws and tangent screw may be used in making this pointing, as it is not essential that the telescope be level.

Rotate the telescope 180° about its axis and correct one-half the apparent error, if any, by adjusting the reticule. Adjust one wire at a time until the intersection will remain on the point throughout the entire revolution.

The point used for this adjustment will preferably be at about the distance of the average length of sight. This adjustment makes the line of collimation coincide with the axis of the rings.

3. *Level.* — (*a*) Lateral. Level approximately and turn the telescope slightly in the Y's. If the bubble remains fixed, the level is in the same plane with the axis of the rings. If not, adjust by the lateral screws at one end of the tube.

(*b*) Vertical. Level up over both pairs of screws, then clamp over one of them. With the clips open, center the bubble accurately. Carefully remove the telescope from the Y's and reverse and return it. Correct one-half the apparent error, if any, by the use of the adjusting screws. Relevel and repeat.

This makes the level parallel with the bottom element of the rings and, if the rings are of equal size, with the line of collimation. This is therefore an indirect or mechanical way of making the line of collimation parallel with the level and, especially with an unknown instrument, it should be checked by the direct or peg method.

4. *Y's.* — For convenience the Y's should be adjusted to nearly the same length, so that the level tube will be perpen-

dicular to the axis of rotation. Level over both pairs of screws; finally over one and swing 180° in azimuth. Correct one-half the error, if any, by adjusting the Y's.

c. Dumpty Level Adjustments.

1. *Horizontal Wire.* — The same as for the Y level.

2. *Level.* — The level is made perpendicular to the axis of rotation by the same method as was used for the Y's of the Y-level, except that the level is adjusted, as the standards supporting the telescope are usually not adjustable.

3. *Line of Collimation.* — Use the two-peg method, adjusting the wire instead of the bubble as in the transit.

The above are the ordinary methods of adjustment which assume that the line of collimation is a fixed line. Since this line is determined by the wires and the optical center of the objective, its direction is not constant for different lengths of sight unless it is parallel with the object glass slide.

To test this for the horizontal wire, as in leveling, the three-peg method is necessary. Select three points at about the same distance apart and determine the exact differences of elevation by taking readings for each pair from a point halfway between. Then setting up near one of the end points, take readings on all three. If the differences are the same as before, or if the discrepancies are proportional to the distances from the instrument, the objective slide is parallel to the line of collimation; if not, the slide may be adjusted on some instruments, while on others the only remedy is to return the instrument to the maker.

For testing the vertical wire of the transit, line in a series of pegs, making the first one 30 to 50 feet from the transit. These points should be in a straight line. Test by plunging and reversing, sighting on the first point, and seeing if the others are on line.

PROBLEMS.

1. On a certain line it is decided to elevate the outer rail on curves at the rate of $\frac{5}{8}$ of an inch per degree of curve.

a. Find the chord whose middle ordinate will equal the required elevation.

b. For what speed will this elevation be correct?

2. Find, by an exact method, the middle ordinates for curving 32-foot rails to 20, 35 and 50 degree curves and compare your results with Table XII.

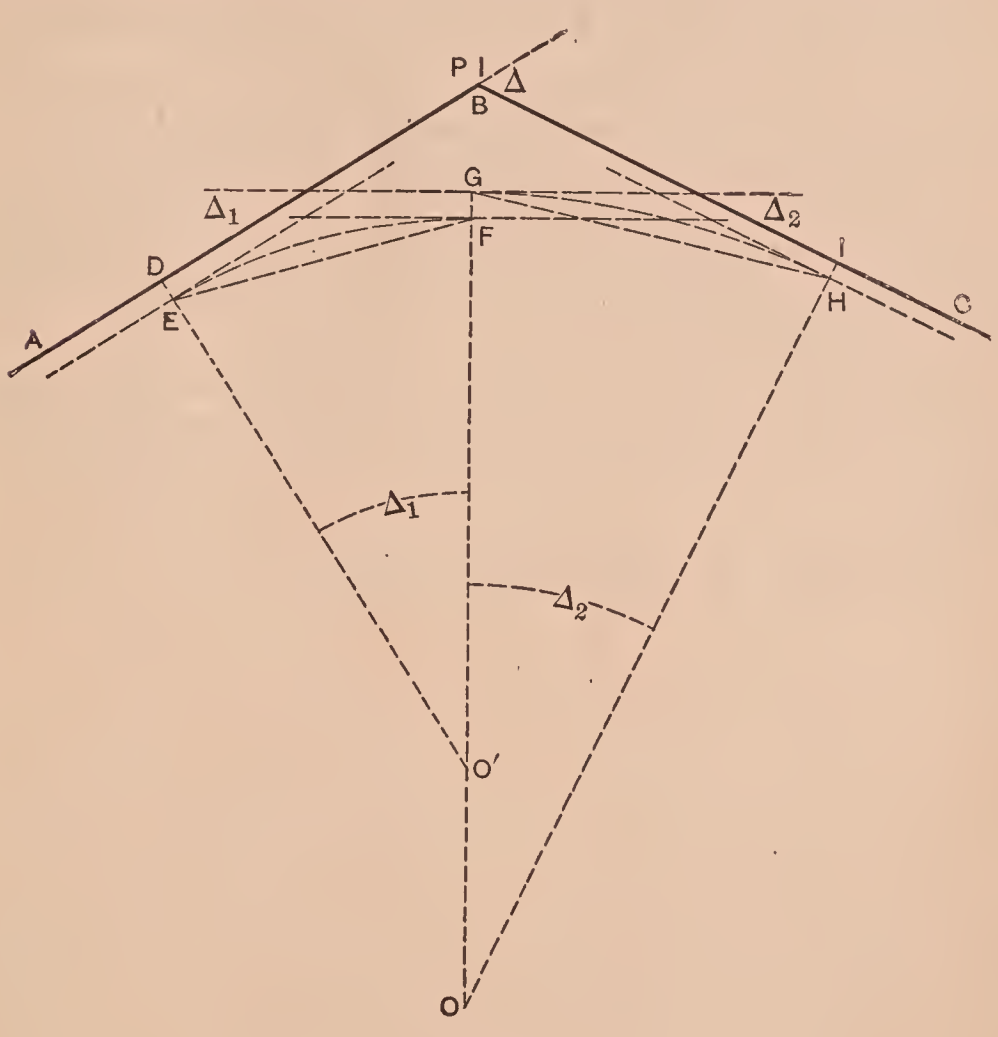


Fig. 23.

3. A track on 3-foot gage is to be elevated for a speed of 20 miles per hour.

a. Find the elevation in inches per degree of curve.

b. Find the chord whose middle ordinate equals this elevation.

4. A + 0.3 per cent gradient intersects a —0.7 per cent at sta. 424 + 50, with an elevation of 478.50. Find the grades at the regular stations for a vertical curve changing gradient at the rate of 0.1 per cent per station.

5. Given the data of problem 4. Find the grades at the regular stations and + 50's for a vertical curve changing gradient at the rate of 0.1 per cent per 50 feet.

6. A 6° curve is elevated for a speed of 45 miles per hour. Find the length of transition curve required by the recommendations of the Am. Ry. Eng. and M. of W. Assoc.

7. How much is the track shortened by the application of 300-foot transitions to a 6° curve 6 stations long?

8. Find the tangent distances if a 300-foot transition be added at one end only of the curve of problem 6.

9. Compute coördinates for points 60 feet apart on a 360-foot transition to a 9° curve.

10. Apply formula (14) of Part II to the determination of the deflections for running in the transition of problem 8 from the PTC, from the PC_1 and for each half from its middle point.

11. In Fig. 23, Δ_1 is 28° and Δ_2, 32°. The degree of the first curve is $D_1 = 9°$ and of the second, $D_2 = 5°$. Find the tangent distances if a 180-foot transition is used at the beginning of the 9° curve, a 160-foot between the two curves and a 250-foot at the end of the 5° curve.

SUGGESTION. — Use the offsets and long chords as the courses of a traverse in computing the coördinates of I referred to the first tangent as an axis and D as an origin, *i.e.*, compute $DEFGHI$ as a traverse.

12. Compute the tangents for the compound curve of the preceding problem if no transition curves are used.

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